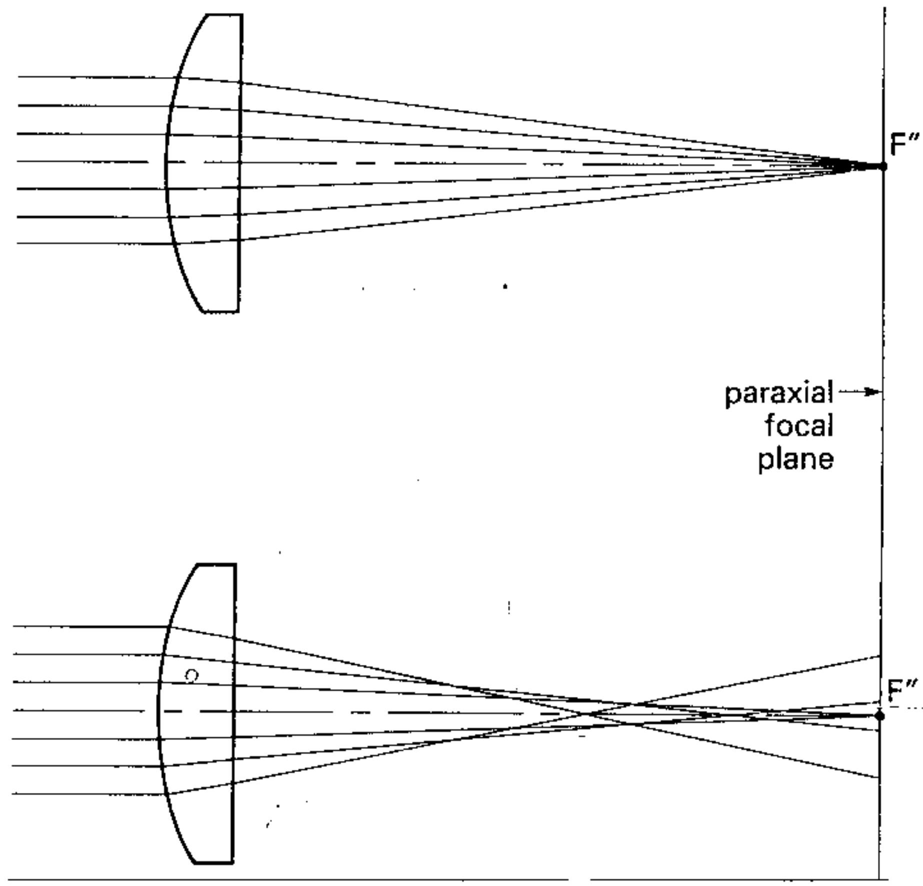
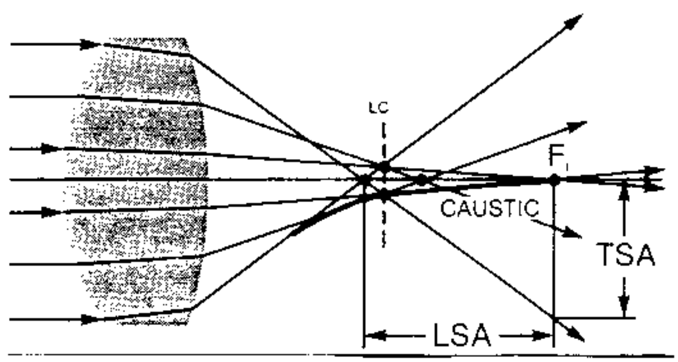
THE SEVEN DEADLY ABERRATIONS
#1: SPHERICAL ABERRATION

EXPLANATION: Due to the geometry of spherical surfaces, rays from a particular point on an object passing through the edge of a lens focus nearer to the lens than do the paraxial rays.

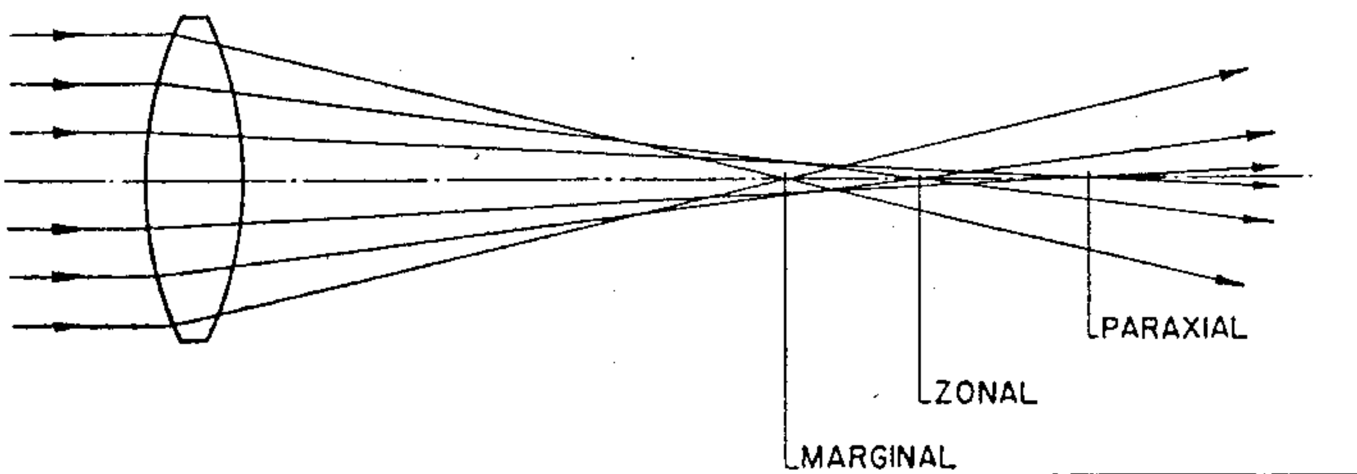


In the top half of Figure 1 above, the ideal ray trace shows that all rays from the same infinite object point arrive at the same focal point. Below that is shown the typical spherically aberrated case, where the marginal rays from the edge of the lens come to a focus nearer to the lens than the centrally located paraxial rays.

This is simply a consequence of applying Snell's Law to all the points of a spherically curved surface; the rays hitting the peripheries of the lens are incident at larger and larger angles, and the beams will be bent much too strongly to match those coming in through the center.



The extent of SA can be measured linearly in two dimensions: along the optical axis, the <u>Longitudinal Spherical Aberration</u>, which is the distance from the paraxial focus to the focal point of the marginal rays; or at right angles to the optical axis, the <u>Transverse Spherical Aberration</u>, which is measured in the focal plane containng the paraxial focus to where the marginal ray intercepts the focal plane. The "miss" angle between the actual path of the marginal ray and its intended path to the paraxial focus is another possible measure of the degree of SA.



An outline of the rays leaving the lens and headed to their respective foci is labelled the <u>caustic curve</u> in Figure 3. Where the marginal rays cross the caustic is the smallest circle of confusion, as after that the marginal rays are expanding the diameter of the circle of confusion. The image at the paraxial focus will look like a bright nucleus with a hazy halo thanks to the marginal rays; the circle of least confusion will have maximum contrast and will be a small patch of light.

Since the marginal rays define the position of the smallest circle of confusion, the position of best focus will change as a spherically aberrated lens is stopped down. The new marginal rays defined by the aperture meet the caustic closer to the paraxial focus, so it seems that the object has moved closer to the lens since the new focal plane is further from the lens.

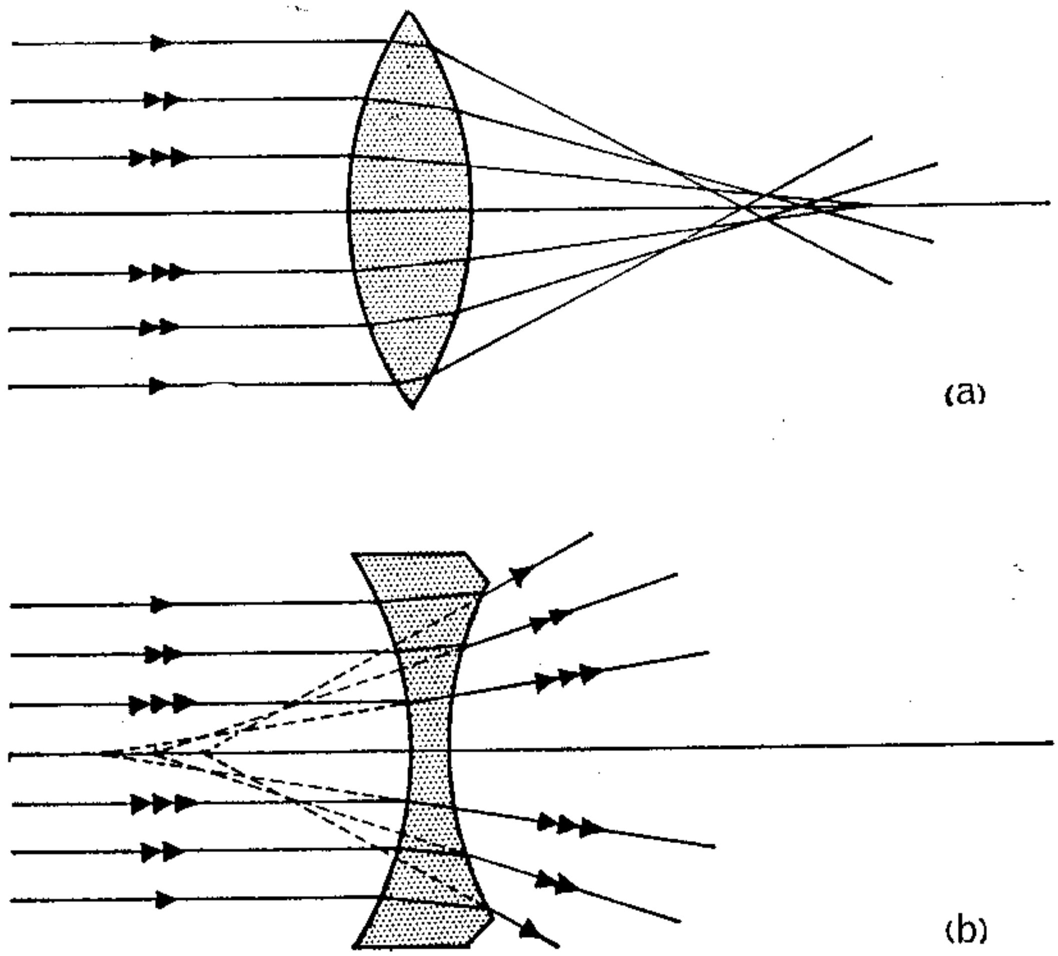
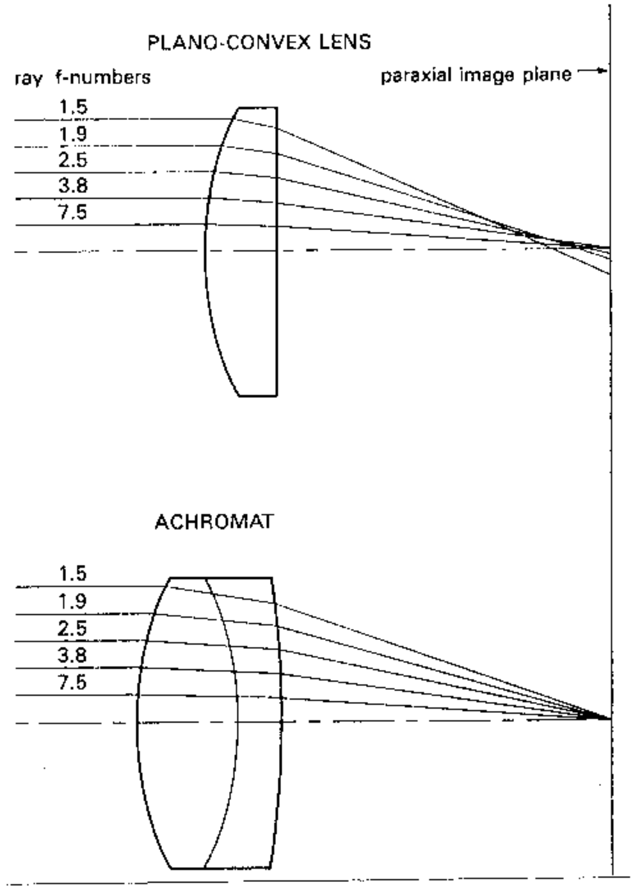


Figure 4 compares the SA of positive and negative lenses. Notice that in both cases the marginal rays bend quicker than paraxial rays so that the marginal focal point is shifted closer to the lens. This makes the marginal rays more convergent or more divergent. When the marginal rays are inside the paraxial ones, this condition is termed undercorrected spherical aberration; the "outside" divergent rays of the negative lens are said to be overcorrected.

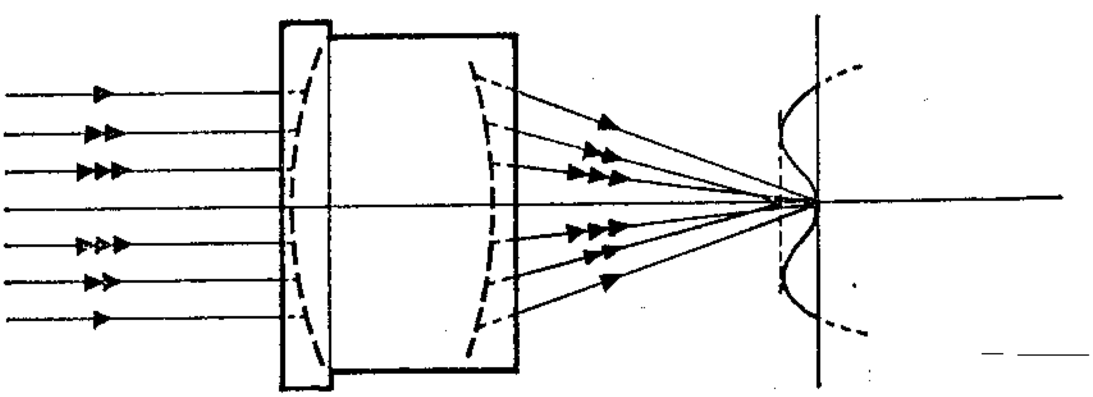
It can be predicted from the figure above, that when observing SA in the virtual image of a negative lens, the image spot will move toward the lens as the point of observation moves further off-axis. The reverse happens in the real image of a positive lens, where the marginal rays move the image closer to the lens, away from the viewer.

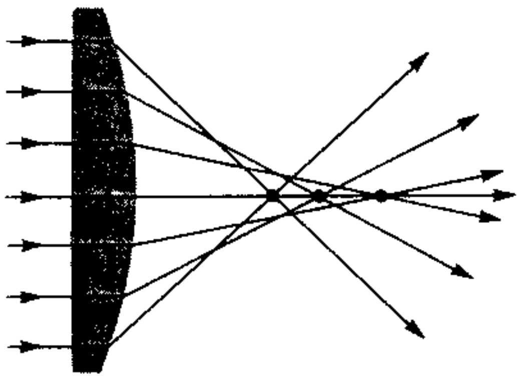
It would seem that a doublet of positive and negative lenses could counteract the SA of each other, and this is a technique used to eliminate SA. Figure 5 shows how much better a cemented achromatic doublet performs than a single element of comparable f/# and focal length, plus correcting for <u>Chromatic Aberrations</u>. (See the **Aberration Handout, #6 & 7.**)

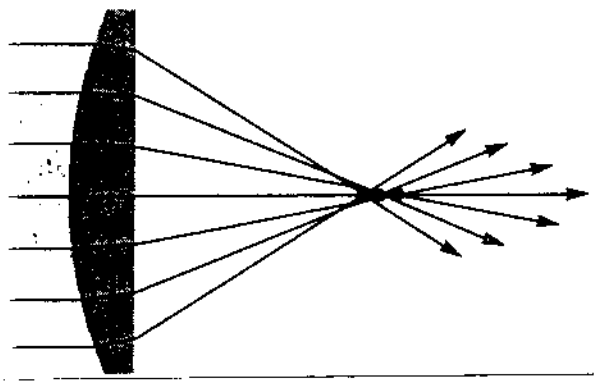


The most obvious cure to minimize Spherical Aberration for a given lens is to stop it down, to get rid of the marginal rays. Of course, this dims the image. Since SA varies with the area of the lens that is transmitting the light, like f/stops, its damage is cut in half by stopping down from f/8 to f/11. Commercial photographic optics are so well-corrected that even at wide open the SA is undetectable to the unaided eye.

The danger correcting the Spherical Aberration of one lens with another is that they may not exactly cancel out each other. Figure 5 shows how the SA may wander. The marginal rays focus closer to the paraxial rays than the middle rays.





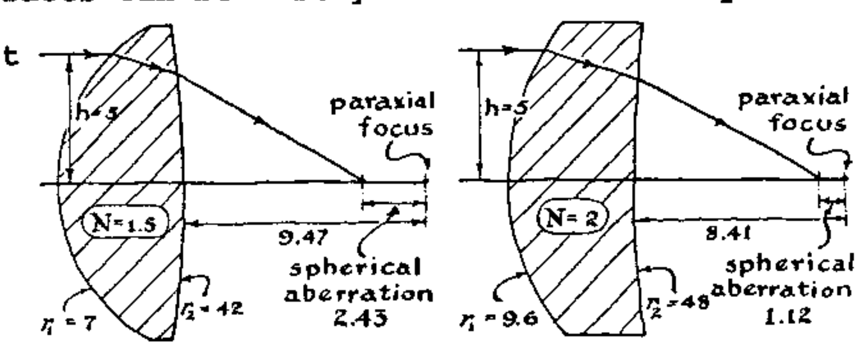


Proper orientation of the lens can minimize SA. Figure 6 shows the SA for a Plano-Convex lens with the flat side toward or away from the collimated rays. In the top case, all bending occurs as the light exits at the curved surface.

With the flat side toward the focus, light is refracted first at the curved surface, and the now non-parallel rays are further refracted by the flat side. By distributing the bending chores between the surfaces there is less mis-aiming of the rays. The basic rule of thumb is therefore "Flat(ter) side toward nearer focus."

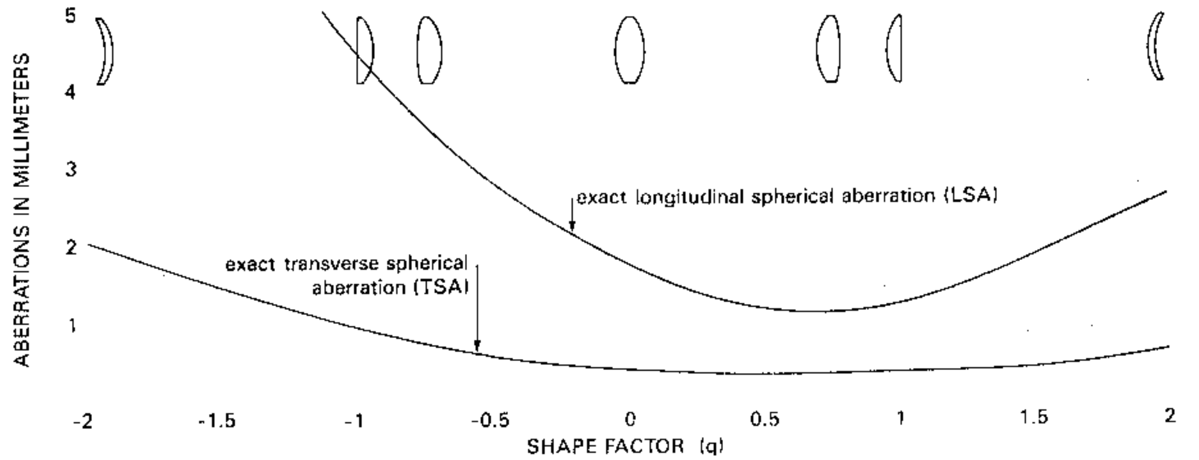
Further examination of Figure 6 shows that if the PCX lens is used as a collimator, with a point source of light at the paraxial focus, the marginal rays will come out not parallel but convergent. The edge of the lens focusses parallel rays to a point in front of the paraxial focus; to get parallel rays out of the edge of the lens the source should be at the marginal focal point. But since the paraxial source is behind the marginal focal point, the edge rays will converge to a focus, however distant. Laser beam collimators usually work at rather high f/#'s to alleviate this condition.

To minimize SA with spherically ground surfaces, the curves of the front and back surfaces can be wisely chosen. The examples in Figure 7 show the best prescriptions that can be devised for telescope objectives, which work at infinite conjugate ratios. In the shapes for two different refractive index glasses, notice that one is double convex, although not



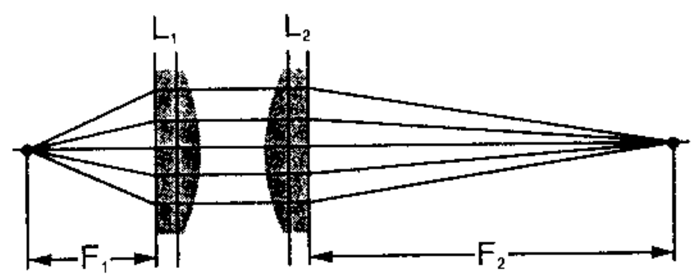
symmetrical, and the other one is meniscus. Since the r2 of each lens is rather long compared to the front surface, it appears the closest match to the best form from off the shelf optics for this application would be Plano-Convex.

A comparison of different lens shapes and their Spherical Aberration at infinite conjugate ratio is shown in Figure 8 below. The <u>SHAPE FACTOR</u> relates the the ratio of the curvatures of the front and back surfaces; Shape Factor 0 is symmetric Double Convex, and as the Shape Factor number increases, r2 goes from convex to flat (shape factor 1) to concave, making a meniscus lens. Once again, the minimum TSA and LSA occur with an almost Plano-Convex Lens.



For any particular fixed conjugate ratio, there is always a best form for a single element to relieve SA. See the next **Aberration Handout, #2, COMA**, for a discussion of the <u>Abbe sine condition</u>, which eliminates both SA and coma for a fixed position. The lens will work well in the neigborhood of that conjugate ratio, but may not be very good at other positions.

Custom fabricated optics can be prohibitively costly. For some applications a pair of PCX lenses can give decent results, configured as in Figure 8, as both lenses are used in their best form orientation.

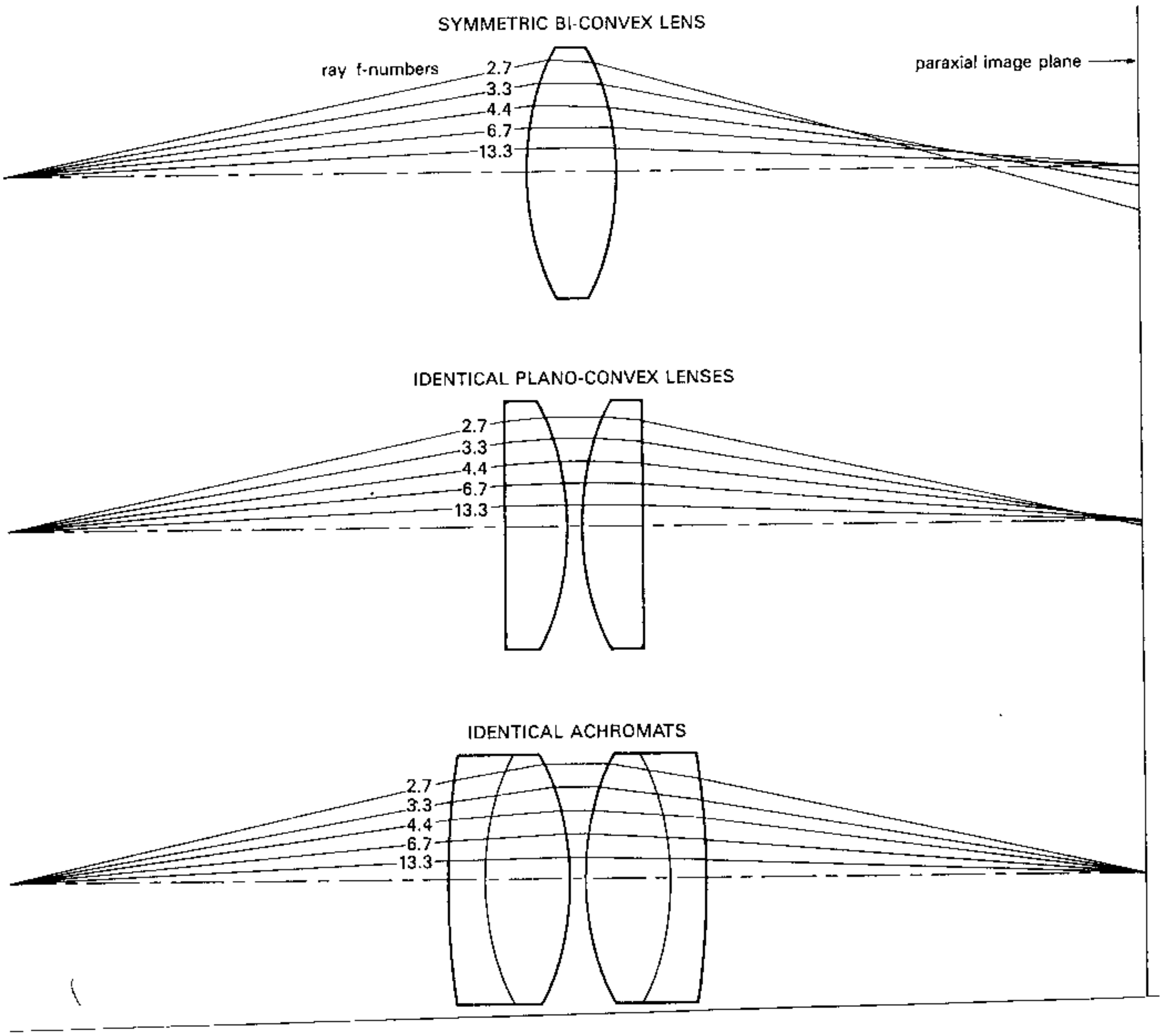


An overhead projector objective is fabricated along these lines. A moderate focal length PCX lens is placed one focal length away from the transparency. This working distance includes enough room for the presenter to write on the sheet without having the optics head in the way. The parallel rays out of the first lens are collected with a second, longer focal length lens that

focusses the light onto the screen, placed the second lens' focal length away.

At the other extreme of conjugate ratios, one to one imaging, where the object and image are both the same distance from the lens, a Double Convex lens is the best form for single elements in this application as shown in Figure 9.

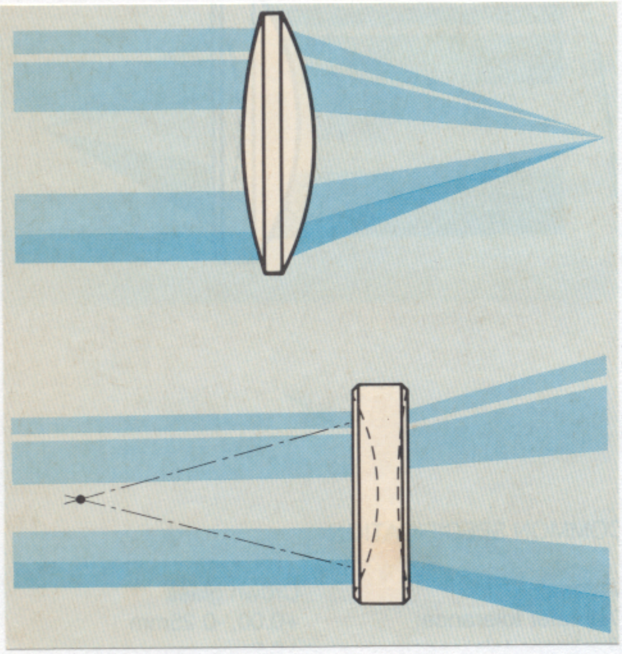
Further improvemnt in SA is afforded by a pair of twin PCX with their curved surfaces toward each other. Both are used at infinite conjugate ratios, for which they are best-form, and since the refracting job is spread between four surfaces, there is less misdirecting of the marginal rays.



Achromatic doublets exhibit much less Spherical Aberration for all f/# rays than does a similar focal length and diameter Plano-Convex Lens, as was seen in Figure 5 above, and a pair of them would give the optimum performance. This configuration is the basis for many different types of lenses, most notably the **Rapid Rectilinear** 19th century photographic lenses. For with the stop in the center of the two lenses, Coma and Distortion are minimized along with the Spherical and Chromatic aberrations. (See the **Aberration Handouts, #2 and #5**.)

To make a lens that will focus parallel beams to a perfect spot requires a surface whose curvature varies from the center to the edge. Since the surface described is not a sphere, a lens built like this is termed aspheric.

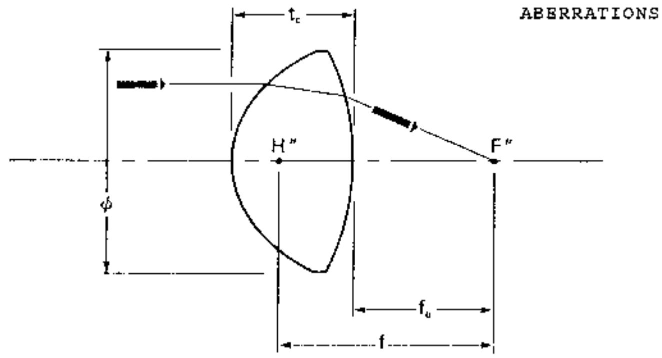
When designing a lens system, complete elimination of the spherical aberration can be accomplished by the use of aspherics. These types of surfaces are much more expensive to grind than the typical spherical ones, as they are turned on a lathe with a diamond tool instead of being ground.



Focussing a laser beam to the smallest possible spot requires complete elimination of spherical aberration, and Figure 10 shows single element aspheric lenses designed for that purpose. A negative aspheric is shown along with the positive asphere. Some lasers are so intense, that they can create unwanted sparks from breakdown of the air at a real focus. To diverge these giant pulsed beams like that of a Ruby laser for holographic purposes, a negative lens with a virtual focus will spread the beam without the danger. These optics are fairly expensive, with prices over $100 apiece.

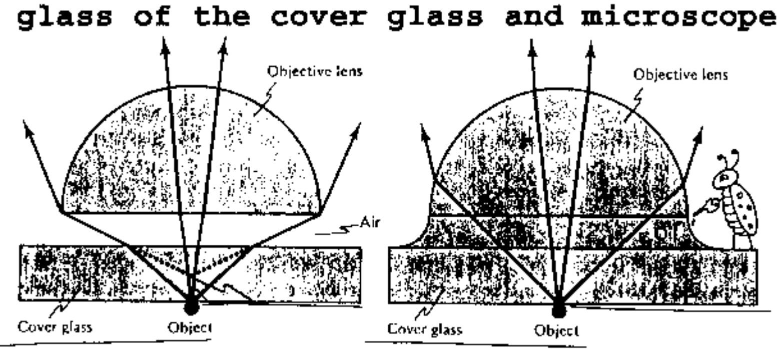
The further the rays are from the optical axis, the lower f/# rays, the closer they focus to the lens. Applications requiring very low f/#'s, like condensing lenses also use aspheric surfaces to make the most efficient collection of the light. An f/.63 all spherical surfaced lens would have a very long Longitudinal Spherical Aberration; Figure 11 on the next page shows the same f/numbered lens in aspheric form, and its performance is markedly better.

These aspheric condensing lenses are not as expensive as the laser line focussing lenses as their surfaces are not as well-figured, being molded and then just felt-polished, since they are used in a white-light application and are not part of an image forming objective.



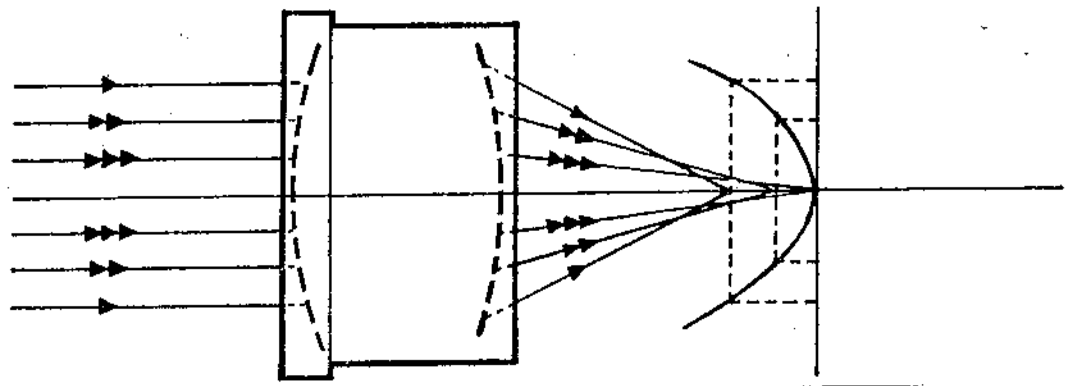
Although the name spherical aberration seems to imply that the aberration is a result of spherical surfaces, the same kind of deviation of rays can take place in any type of optic. For instance think of the rays diverging from a laser pinhole spatial filter point source and entering a cubic beamsplitter or combiner as in Figure 12. At the first surface refraction takes place, then the exiting rays refract further outward. But when the edge rays are traced backwards to their source, they seem to come from a different position than the central rays. Same problem as with spherical lenses, except all surfaces are planar.

A similar effect occurs in microscopy, as illustrated in Figure 13. A flat cover glass, about 100 microns thick, on top of the specimen can spherically aberrate its image. An oil of the same refractive index as the glass of the cover glass and microscope objective couples the light between them without refraction, so that the image is sharper. These **Oil-Immersion Objectives** can provide magnifications of 85 - 100X.

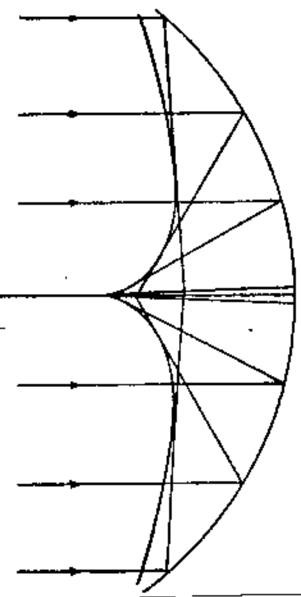


Sometimes this aberration is entitled **Aperture Aberration**, since the misbending is a function of which part of the opening the light enters and exits from.

The Longitudinal Spherical Aberration is a function of how far from the optical axis the rays exit the optic. Figure 14 graphs the position of the foci versus the f/# ray. The vertical axis is the ray f/#; horizontal is the focus position.



Notice that the graph is a parabola, since LSA is proportional to (position on the diameter) squared, just like f/stops. Stopping down alleviates LSA. Transverse SA is dependent on the cube of the distance from the optical axis.



Spherical Aberration can occur in reflecting optics, too, like in Figure 14. The marginal rays focus closer to the mirror than the paraxial rays. Telescope mirrors are typically ground parabolic for this reason, an aspheric solution, although that in itself does not save them from off-axis aberrations like Coma, Astigmatism and Distortion. But since mirrors are not dispersive, Chromatic Aberration is absent.

It was Spherical Aberration that killed the Hubble telescope. The edges of the primary mirror were not fabricated with the proper curvature, and the marginal rays fuzzed out the stellar images.

The **CURES** for Spherical Aberration are:

Proper orientation of the optic, as in "Flat(ter) side toward nearer focus".

Stop down to eliminate Marginal Rays.

Pick the right shapes for the surfaces, either spherical or aspheric. But best-form works best at one certain conjugate ratio.

Use doublets or more elements to have the over-correction of one overcome the under-correction of the other.

SPHERICAL ABBERATION DEMONSTRATION

1. With BBO, show the miss angles of the marginal rays. Identify the LSA and TSA.

2. Show the effect of stopping down with the BBO.

3. Send a Spatially Filtered beam onto the Big Lens and watch the light converge. Show the extent of the LSA by passing the Groundglass through the image planes.

4. Observe the real image of the pinhole with the naked eye. Watch the image point move toward and away from the lens. Do this with a big negative lens, also.

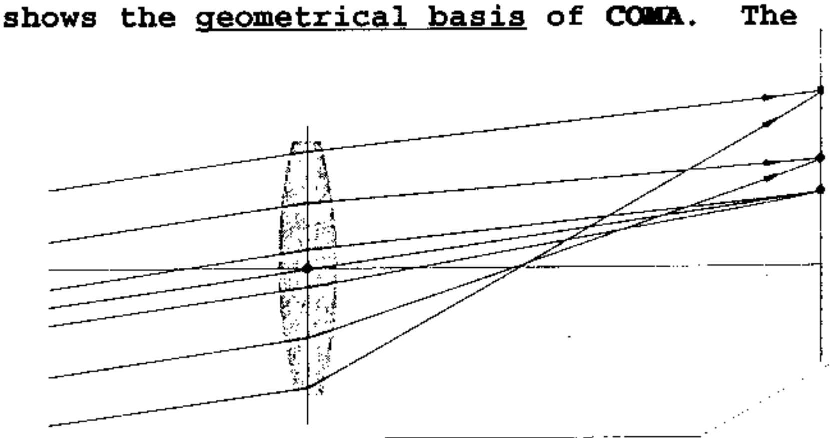
5. Stop down the Big Lens with a variety of Waterhouse stops. Watch the focus change.

6. Place the Hartman Plate on the flat side of the lens and watch the rays come out. Place Groundglass or White Board in the plane of the rays.

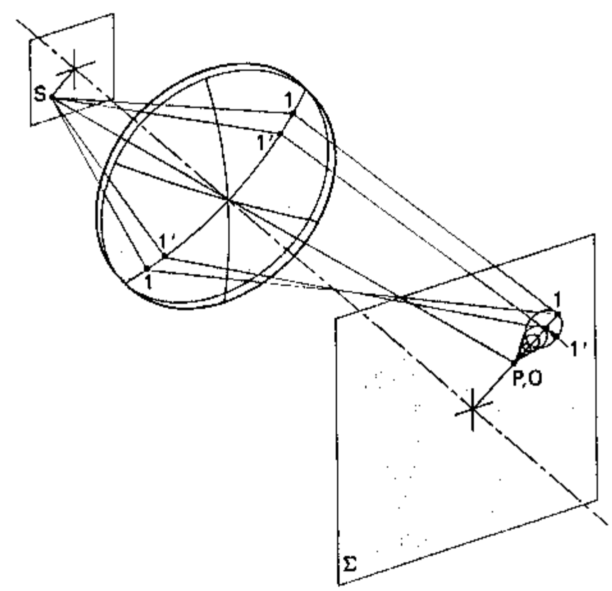
7. Swap the object and image sides of the lens.

THE SEVEN DEADLY ABERRATIONS
#2: COMA

EXPLANATION: Figure 1 shows the <u>geometrical basis</u> of **COMA**. The ray from the object point that passes through the center of the lens defines the principal ray and its focus. More marginal rays focus further off-axis.

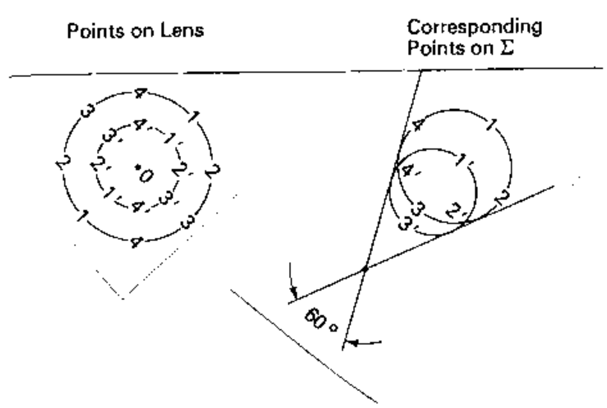


But Figure 1 is only a cross-section of all the other funny stuff that goes on. In the 3-D perspective view below, diametrically opposed points on the same f/# aperture come to a focus at the same spot. However, other points on the same circle don't focus to that same point but onto other points on a circle which contains the image points of all the diametrically opposed members of that same f/# ring.



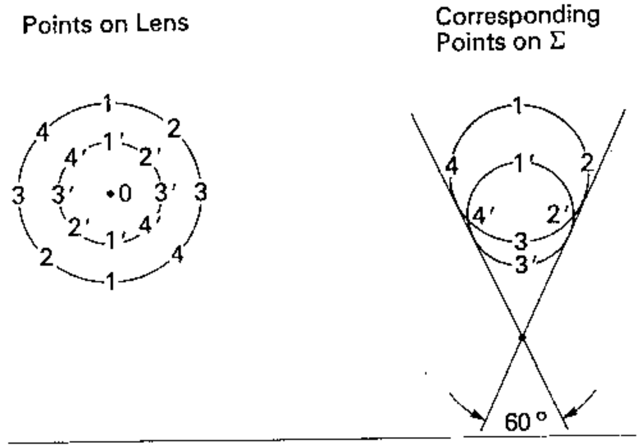
Looking at Figure 2, notice that there is a ray from the object point that goes through the center of the lens. This defines the tip of the comet, P, O.

The circle containing both the 1's (One prime) is at an intermediate f/#. The rays from the object travelling in a plane parallel to the offset of the object point to this ring are refracted to image point 1', away from the paraxial image side of the optical axis. Further from the center of the lens at a lower f/# ring the 1 rays come to focus further away from the optical axis.



From this information it would seem that the image is being smeared into a line. But additional points on the same f/# ring land image rays in a circular pattern. The left of Figure 3 shows the annuli of the two f/stops on the lens, as viewed from the vantage point

of the image plane, with the 1 points labelled as in Figure 2. The right of Figure 3 shows the circular image mapped out as a function of the locations of the rays piercing the lens.



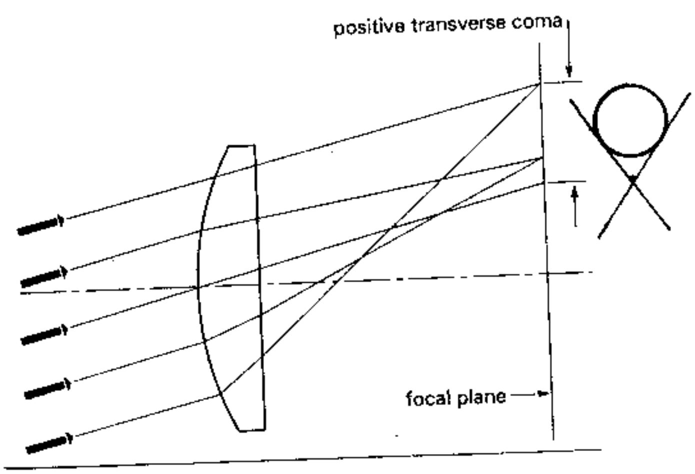
And this generation of the image circle is done in a fishy manner; both point 2s, which are 1/8th of a turn away from points 1, focusses on the image circle 1/4th of a turn from 1s image. The 3 points, 90 degrees away from the 1s, are focussed at the opposite end of the image circle diameter that includes 1.

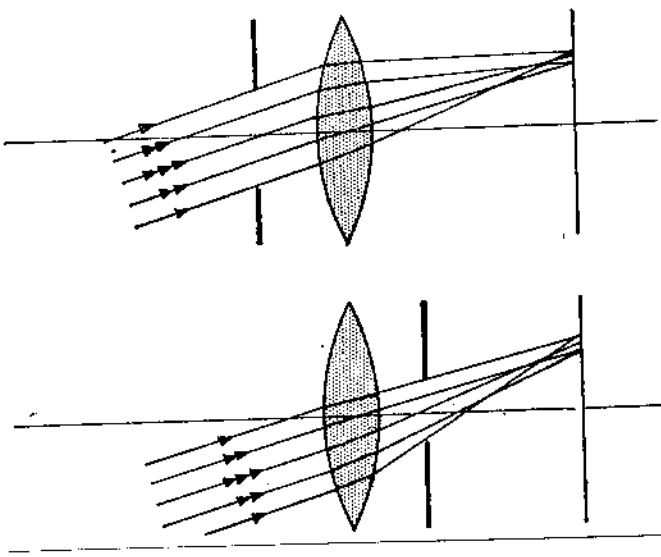
Coma twists the rays around so it is hard to visualize. But if Figure 1 is the cross-section of the plane that includes the two 1 rays in figure 2, and then we shifted to the cross-section that include the 2 rays, the 2 rays focus would be closer to the optical axis, the planar cross-section containing the 3 rays would show the focus even closer to the axis, while the 4 cross-section would move the focus back to the same position as 2.

Every f/# ring on the lens contributes its own comatic circle, so the real comet tail is composed of many overlapping circles. The size and location cause the vertex angle of the comet tip to be 60 degrees.

Coma increases with the square of the distance from the optical axis, and the assymetrical image flare can be the limiting factor in useful field of view of a lens. A stop will reduce coma, like spherical aberration, but its location is critical.

When coma is measured from the principal rays' focus to the far edge of the marginal rays it is termed <u>transverse or tangential coma</u>. By locating the nearest point of that same circle, and measuring its distance from the principal focus, the <u>sagittal coma</u> is found. A bit more than half of the energy of this corrupted image is found in this <u>sagittal crotch</u> in Figure 4.

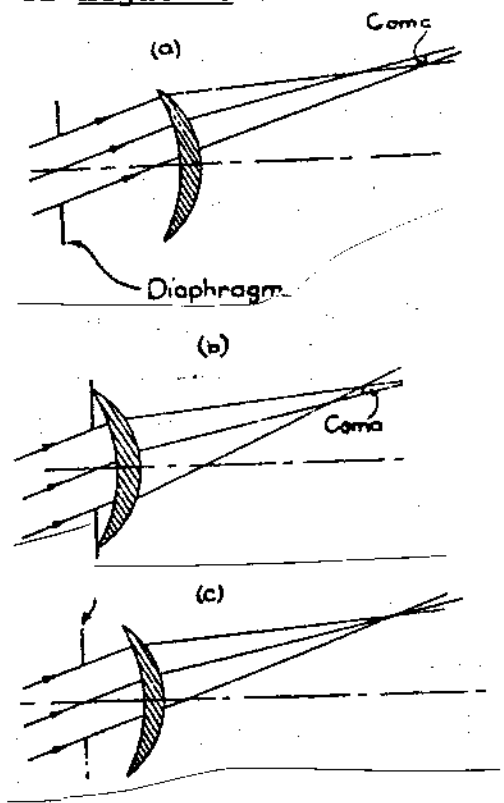


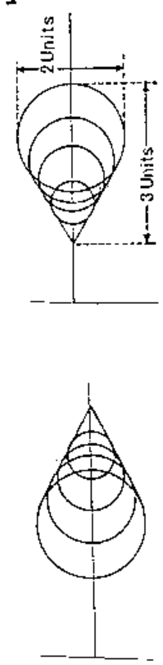


Stops in front or back of the lens can control coma. The comparison takes place in Figure 5. Notice which ray determines the tip of the comet in each case; it's not the central ray through the stop.

Figure 6 below shows the effect of a front stop position on coma. When the stop is far in front of the lens, the comet points toward the optical axis, the coma is dubbed <u>inward</u> or <u>positive</u>.

When the stop is as near as possible, touching the lens, different rays pass along with the principal ray, and now the tip of the comet is away from the optical axis, in <u>outward</u> or <u>negative</u> coma.



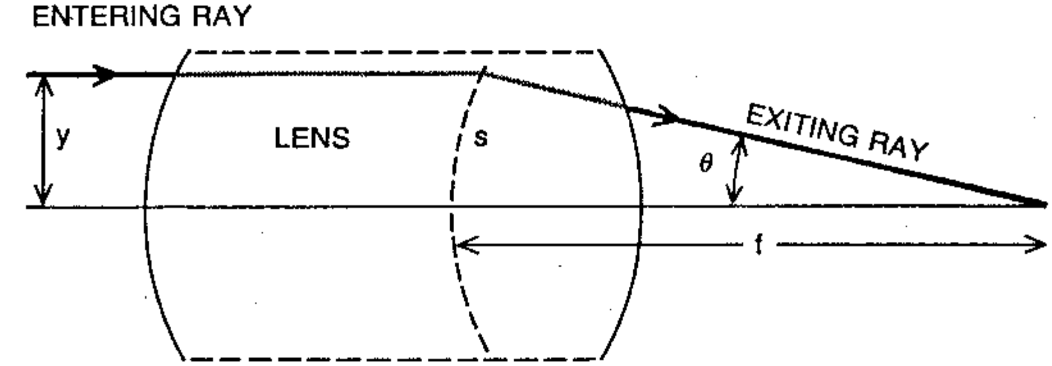


There is a happy medium spot that will allow all the rays to agree about the location of the image spot, eliminating the splayed out comet-shaped blur. Spherical aberration is not aperture position dependent, only size, so the on-axis image points are improved also.

This fact, coupled with the shape of the lens, allowed camera manufacturers to use a single meniscus lens as the objective in low end models for many years

For a certain image-object relationship, there is a shape of the lens that will have zero coma. For example, a plano-convex lens with the flat side toward the focus of a bundle of rays from an object at infinity has almost zero coma. It was low in spherical aberration at this conjugate ratio, too.

For other conjugate ratios other lens shapes could be computed. Figure 7 shows the principle involved, the Abbe Sine Theorem. The lens is a multi-element design, with individual lenses not shown; the ray-trace through the lens should show that such a surface exists. If not, there will be coma.



ABBE SINE CONDITION, formulated by Ernst Abbe, specifies the condition for simultaneous correction of spherical aberration and coma. When the rays entering the lens and the rays leaving the lens are extended, they intersect at a surface, *S*, defined as the locus of the points at which all rays from an infinitely distant point source on the axis of the lens appear to be refracted. The dual correction is achieved when y is equal to $f \cdot \sin\theta$; therefore surface *S* is spherical.

The double PCX trick shown in the **Aberration Handout #1, Figure 8** would work here again using off the shelf optics, if a stop is placed between the two lens elements.

CURE:

Stopping down to very small openings will eliminate coma.
Proper placement of the iris will minimize coma at intermediate f/#'s.
Shape of the lens can bring coma down to zero for a certain image-object distance, thanks to the Abbe Sine Theorem.
Lenses whose comas are opposite in sign can cancel each other out. Symmetric placement on either side of a stop works wonders!

COMA DEMONSTRATONS

1. With BBO, show coma through the lenses and off the mirrors.

2. Show the effect of stopping down with stops in front of and behind the lens with the BBO.

3. Send a Spatially Filtered beam onto the Big Lens at a slight angle and watch the light converge. Show the extent of the coma by focussing onto the Groundglass and viewing the image with an Agfa Loupe.

4. Stop down the Big Lens with a variety of Waterhouse stops, in front of and behind the lens, and watch the coma flare shrink.

5. Place the Toric Zone Stops on the flat side of the lens and watch the rays come out. Place Groundglass or White Board in the plane of the rays. A double Hartman Plate can be used to show the rotation.

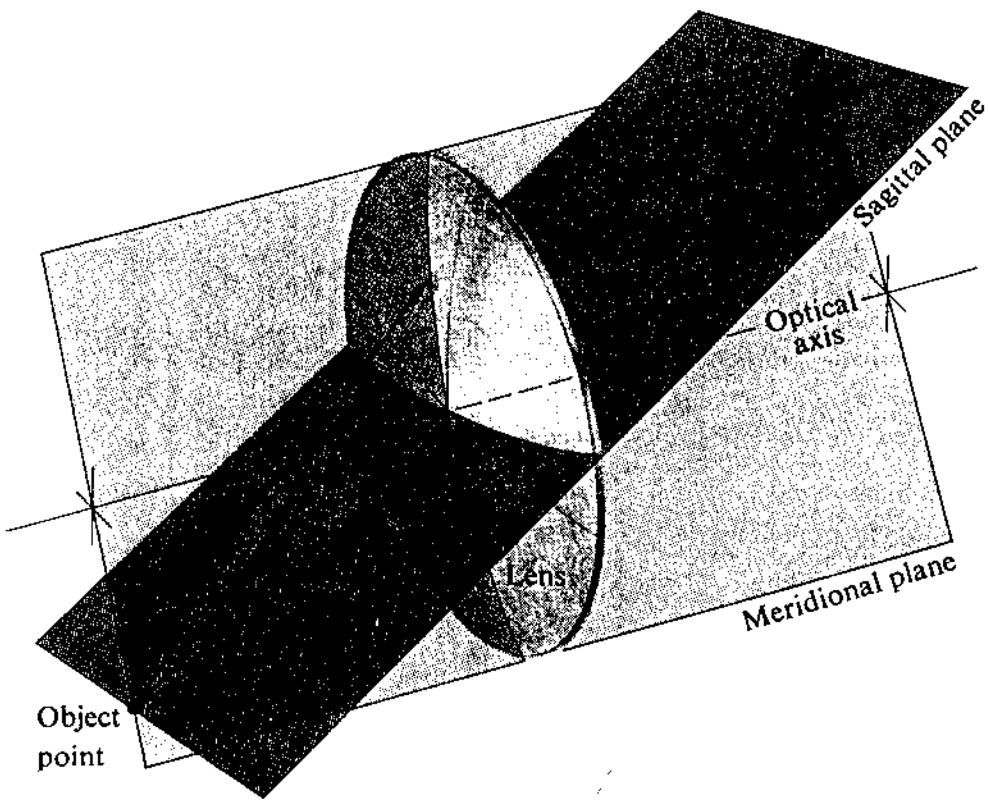
6. Swap the object and image sides of the lens and try some of the tests again.

THE SEVEN DEADLY ABERRATIONS
#3: ASTIGMATISM

EXPLANATION: Spots on the palms of the hand or on the soles of the feet would appear spontaneously on some of the ancient Catholic saints. These stigmata represented the nails of Crucifixion.

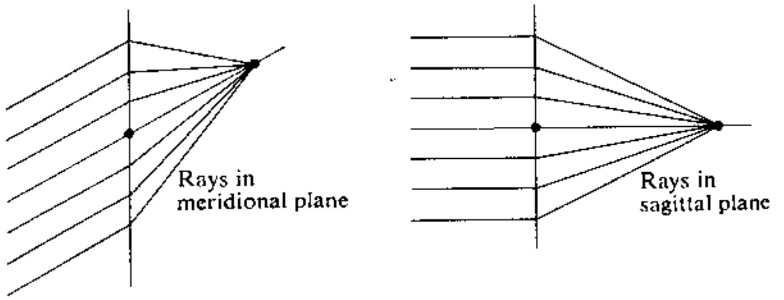
Astigmatism means without a spot. Due to the assymetry of the path of the rays in two orthogonal planes, there are two focal planes for each object point, one in front of the other, so there is no chance of an image point being formed.

Before examining the geometric root of astigatism, it is necessary to define the two orthogonal planes of the ray trace. Figure 1 shows an off-axis object point in front of a lens. There is a ray from the object point passing through the center of the lens and is common to both planes.



The tangential plane (or meridional or radial) also includes the optical axis. It could be thought to lie in the plane of this sheet of paper. The sagittal plane sticks out of the plane of the page, and the optical axis pierces it in the center of the lens.

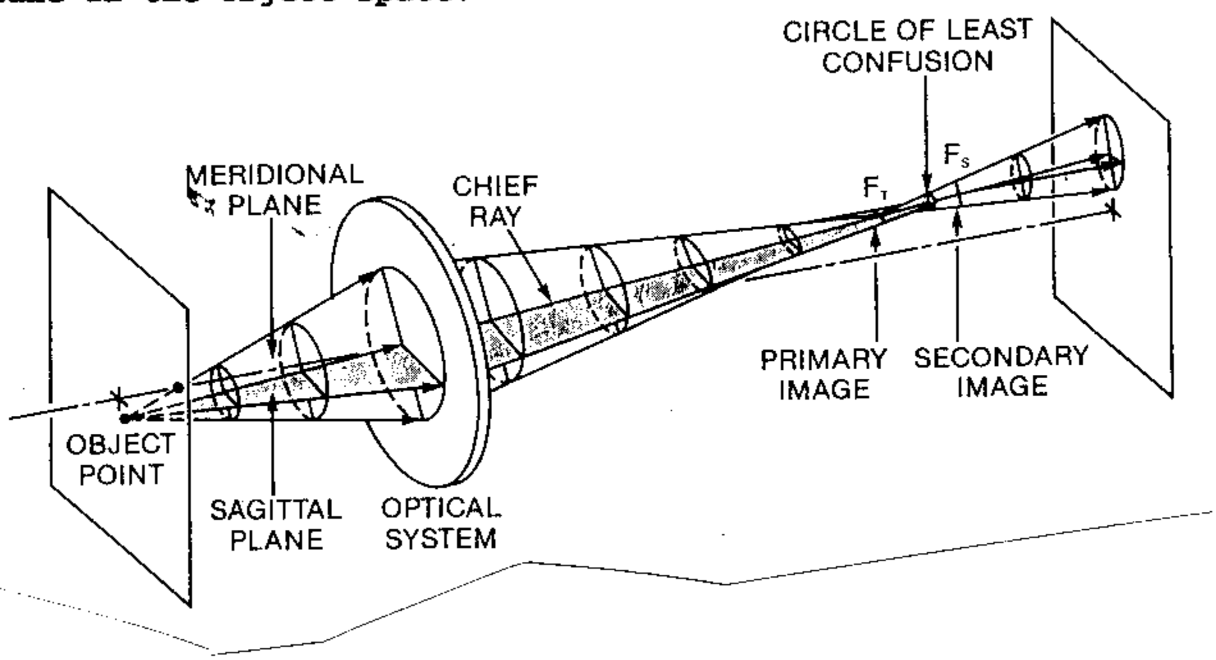
Figure 2 shows the two planes in cross-section. The view of the tangential plane is from the side of the lens in Figure 1. The sagittal rays are viewed from the top of the lens.



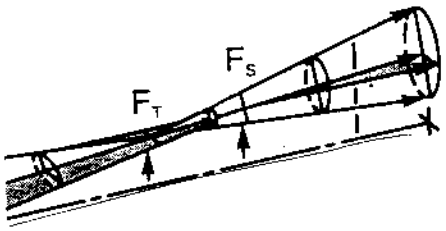
The tangential and sagittal terminology is relative to the position of the object point. If in Figure 1 the object point is considered to be at 6 o'clock, a rotation of the object point to 5 o'clock would tip the tangential plane to extend across the diameter from 11 to 5 o'clock, and the sagittal plane across 8 and 2 o'clock.

With the object directly below the lens, the tangential is a vertical plane and the sagittal is horizontal, albeit tilted. When the object is directly to the side, then the tangential is a horizontal plane and the sagittal is vertical.

Figure 3 is the 3-dimensional construction of the ray trace. As you follow the path of the rays, it is best to think of the tangential plane as represented by a piece of white cardboard, and the sagittal plane by grey cardboard. This explains why the sagittal plane is not seen on the other side of the tangential plane in the object space.



Although the width of both planes is the same at the lens, the white tangential rays are bent more steeply than the grey

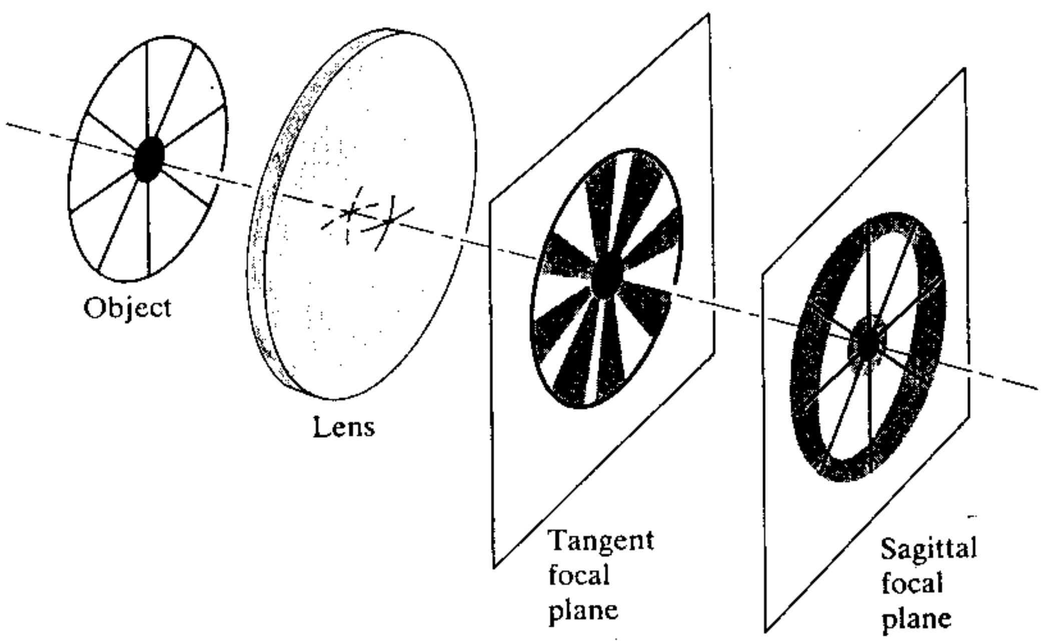


sagittals and focus closer to the lens than the other. Close examination in the neighborhood of the circle of least confusion finds the grey sagittal plane coming into view after the tangential focus, then disappear again behind the divergent rays of the tangential focus. After the sagittal focus, the left and right rays switch sides, and the sliver of grey visible now has crossed over from the hidden side.

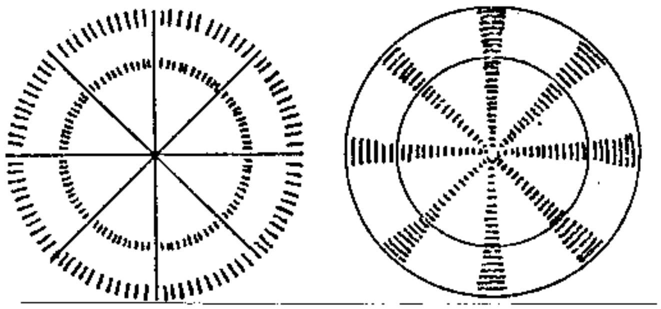
Notice also that although the tangential rays are converging in a vertical plane, this primary image has a horizontal dimension, because the horizontal focus has yet to converge. Similarly the sagittal or horizontal focus is a vertical line, since this secondary image is formed after the vertical rays become divergent.

The line of the secondary image will always point toward the optical axis. As shown in Figure 3, the object point below the optical axis is turned into a vertical line at the secondary focus. If the object point is to the left of the lens, its secondary image line will be horizontal.

Figure 4 shows the interesting predicament of imaging a spoked wheel. Since the spokes are radii, they are resolved by the sagittal rays. Since the circumference of a circle is at right angles to the radius, it is resolved by the tangential rays.



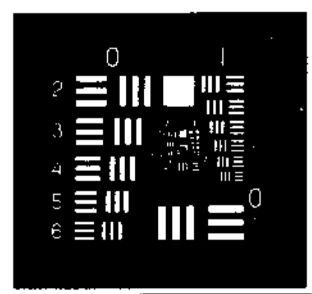
This effect gives a clue as to how the S plane gained its name. The spoke images point to the center, like arrows, which in Latin are sagitta. Everyone is familiar with Sagittarius the Archer in the Zodiac. Notice that there are two t's in sagittal.



In Figure 5 there is a closeup of the two image planes of Figure 4. The sagittal (or radial) focal plane on the left images the radii quite nicely, but the circular lines progressively widen from blur the further from the center of the image. Referring back to Figure 3, the object point is at 6:00, and here its image point is along the 12:00 radius. The groundglass to view the image is placed at Fs, the SECONDARY IMAGE in Figure 3. This is where the grey sagittal plane rays cross horizontally.

By moving the target card forward to the tangential focus, Ft, or PRIMARY IMAGE, the circles become resolved, but the radii broaden outwardly like the arms of a Maltese cross. That's because this plane is intercepting the radial or sagittal rays before they narrow down at their focus.

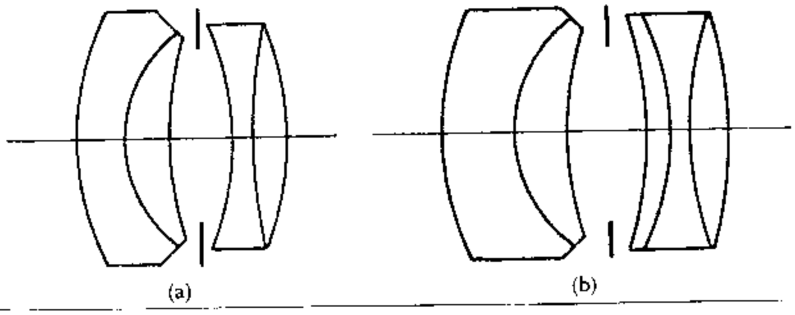
Inspection of the image of the plus signs in the Johnson Lens Tester, (See the **Handout, OPTICAL ENGINEERING NOTE #8, The Johnson Lens Tester**.) shows the degree of astigmatism, and how it can change from the center outward. Astigmatism varies with the square of the angle of incidence off-axis, plus it increases with the square of the image distance; the further away the image, (meaning the closer the object) the more astigmatic. The rate in change of size of the widths of the circles or the flare of the radii in the figure above graph the angle off-axis squared dependence of astigmatism.



The **1951 USAF Resolution Target** in Figure 6 uses alternating bright and dark bands to find the smallest thing a lens can resolve. And it checks the resolution in horizontal and vertical planes by its layout. It is possible in some optical systems, like anamorphic lenses or Rainbow Hologr-ams to have different resolutions in the two planes.

Because there are two focal planes, an object spot is never imaged as a point. The best that can be done is to compromise at a spot along the chief ray's path in between the two foci where a

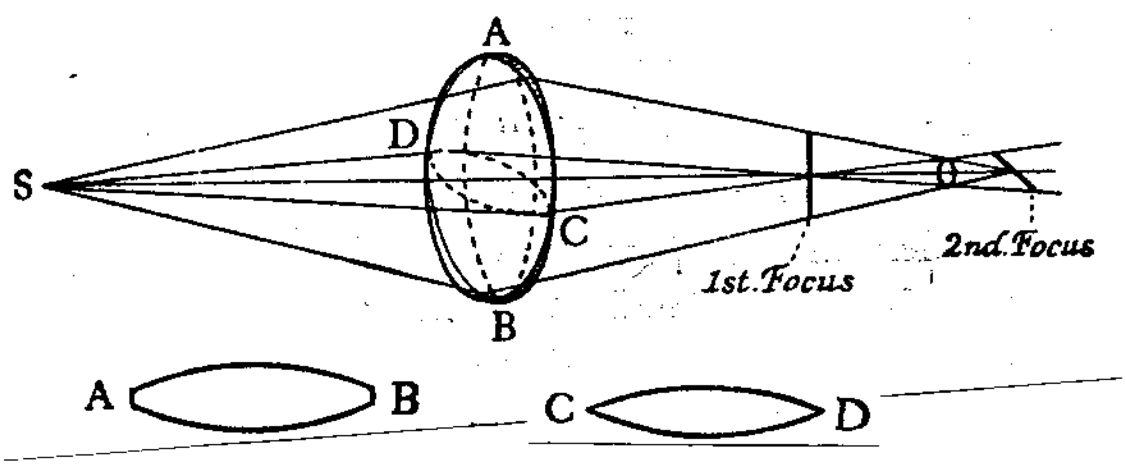
The conquest of astigmatism was the pinnacle of nineteenth century optical enginnering. What had been the missing ingredient in the lens designer's bag of tricks were higher index of refraction glasses. In 1890 the Zeiss Company introduced a series of lenses entitled the "anastigmats", cross-sections of which appear in Figure 10.



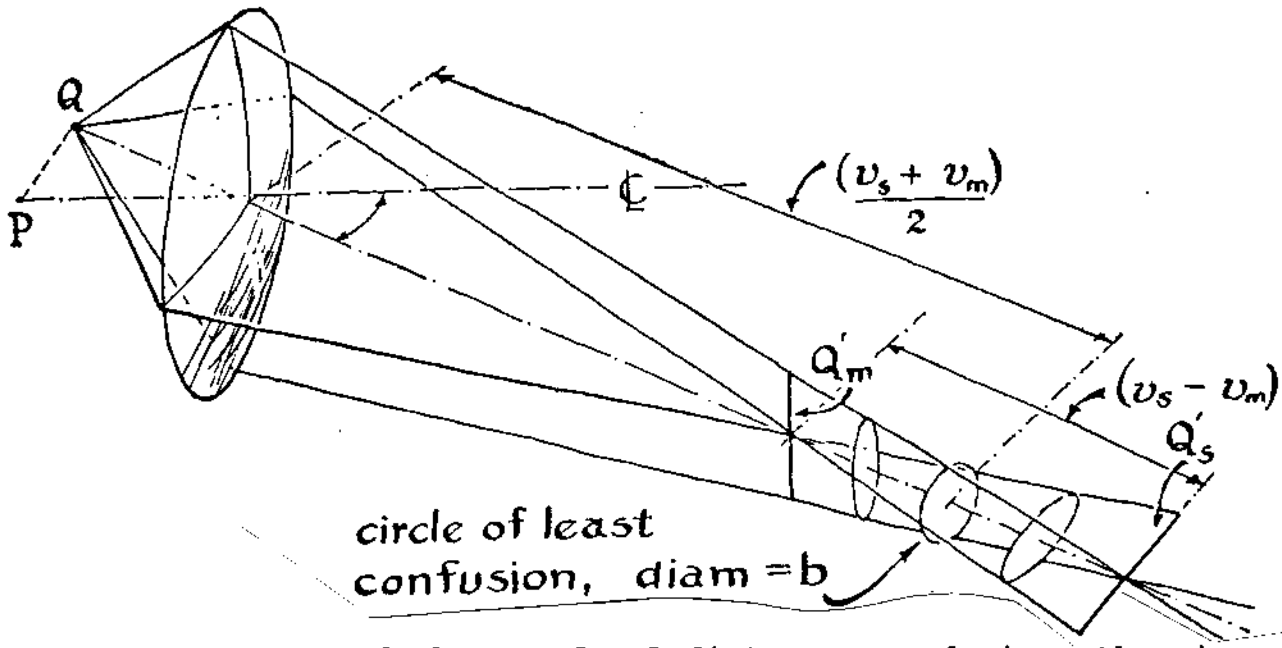
Astigmat means without a point, an-astigmat means without without a point, or stigmatic. Here is an interesting case of a double negative prefix.

Paul Rudolph, the designer of the Anastigmat, considered the prescription to be a pair of achromats, with the front one of the old style of glasses carrying little focussing power but with strong dispersive power, and the rear of higher power with the new, higher index glasses. (b.) shows a three element achromat as the rear element.

Astigmatism of the eye is not an off-axis aberration. It occurs on the eye's optical axis, including the fovea, because the eye lens is defective in shape. Figure 11 shows the cross-sections of the crystalline lens in two perpendicular planes. The radius of curvature of each is different, and then so are the focal lengths. The eye can only accomodate for one focal plane at a time, unless there is enough depth of field, so the afflicted will not see both patterns of Figure 12 (next page) in focus at the same time. Nor will an astigmatic lens. Notice in Figure 11 that the first and second foci are on axis, not off, as in the glass lens case.



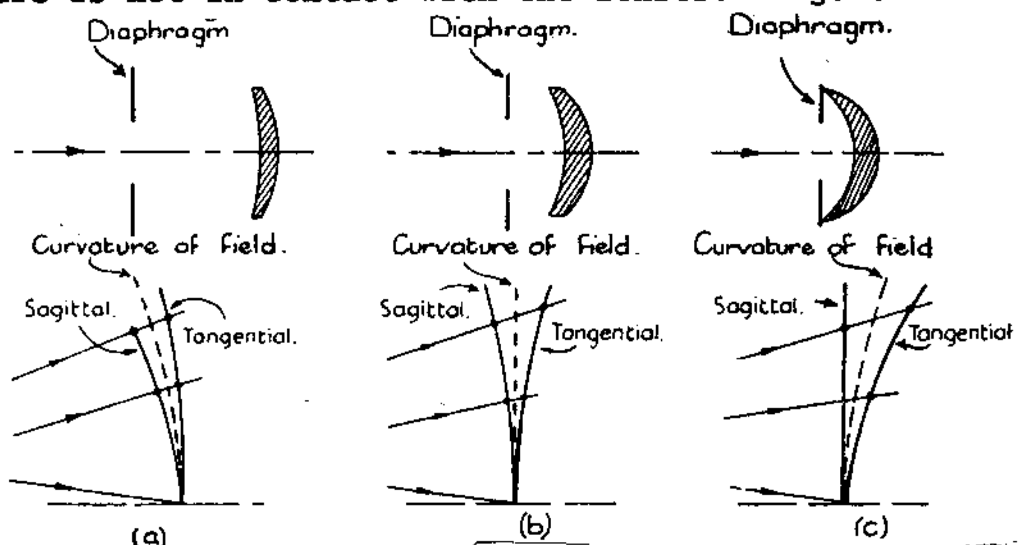
circle of least confusion is formed. (Figure 7.) Its position



is at the average of the two focal distances, and since the size of the two image lines does depend on the aperture, stopping down the lens makes the astigmatic circle of confusion smaller.

The astigmatic difference is the distance between the T and S focal planes. It varies as the square of the distance of the object point from the optical axis, and is not dependent on aperture. Stopping down does not bring the T and S planes any closer together. This dimension is controlled by lens shape, thickness and refractive index.

Astigmatism depends solely on lens shape when the system aperture is not in contact with the lenses. Figure 8 shows the



change in position of the tangential and sagittal focal planes when the lens shape is varied. The diaphragm is located at the best location to eliminate coma. (b) appears to be the best solution, as the plane of the circles of least confusion between the two foci is flat. (This is one way of dealing with **Curvature of Field**, the next aberration to be studied.) Note that the separation between the two planes, the <u>astigmatic difference</u>, increases as the rays move further off axis.

As the rays become ever more oblique, the astigmatism gets even more severe and the sagittal and tangential planes may even wiggle and jiggle back and forth of the focal plane, like in Figure 9. (a.) shows this condition, where the astigmatic difference varies from 0 to a maximum and back to 0 again, with the S and T planes even switching locations. Usually lens designers cure astigmatism only as far as necessary, just to the end of the intended field of view. This is why it is not good practice to use lenses from smaller formats on larger ones. A 50mm lens is normal for 35 mm format, 24 by 36mm. When used on a 2 1/4" square negative camera, 50mm is wide angle. But the aberrations of the rays coming in wider than the normal field of view of the 35mm format would make it unusable as a wide field lens on the bigger format. Wide angle lenses are designed for a particular format and have corrections engineered in for the extremities. Of course, the 50mm wide angle for 2 1/4" would work quite well as a normal lens for the 35mm camera, if it could be made to fit.

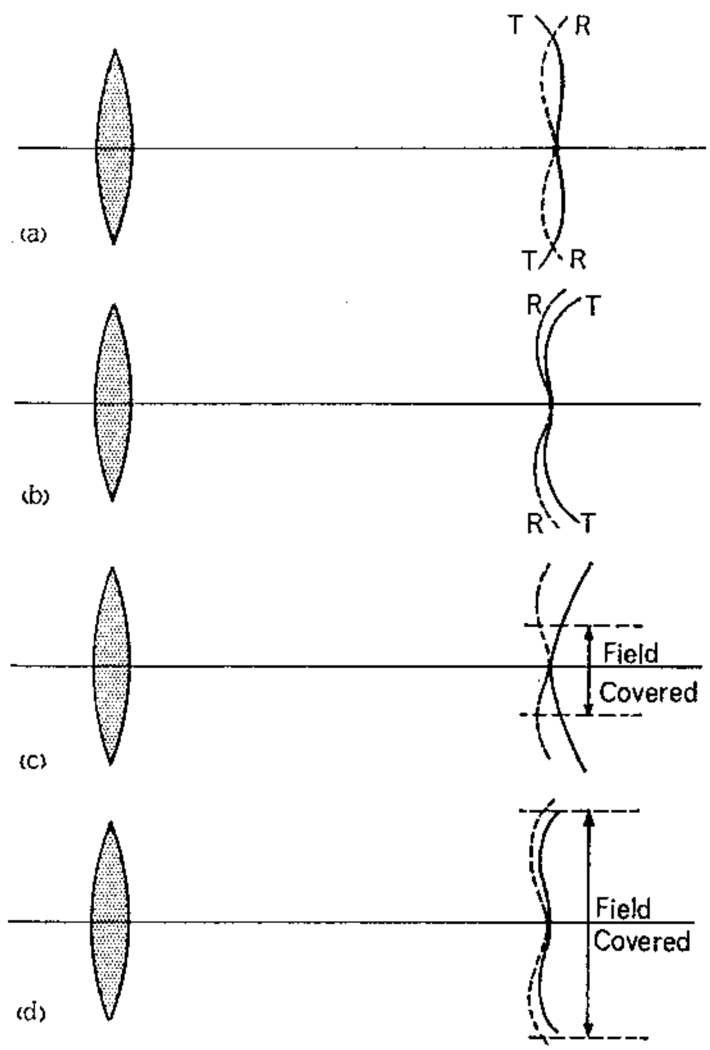
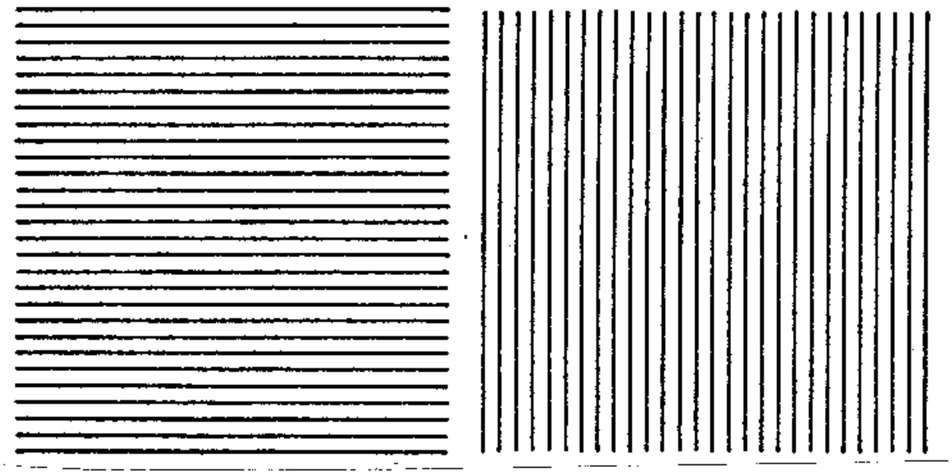


Fig. 35. With images that lie further away from the lens axis the astigmatic focal surfaces may fold back and take the shapes whose cross-sections are shown in (a) and (b). These astigmatic surfaces are adjusted for a narrow angle in (c), and for a wider angle in (d).



ASTIGMATISM DEMONSTRATIONS

1. Send a Spatially Filtered beam onto the Big Lens at a big angle and watch the light converge in the two planes. Show the extent of the astigmatism by focussing onto the Groundglass and viewing the image with an Agfa Loupe.

2. Map out the shape of the tangential and sagittal foci with a tall groundglass on a stand with a line pointng down to some graph paper.

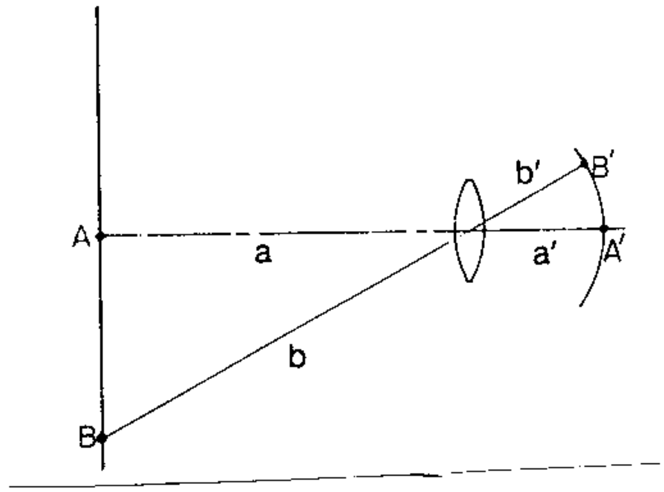
3. On an optical rail, use 100mm lens to image chicken wire, with lens tilted at 45 degrees to induce astigmatism.

4. Focus image of radial test target to see the spoke paradox.

5. Look at the test for astigmatism on the Aberration sheet through a cylindrical lens.

THE SEVEN DEADLY ABERRATIONS
#4: CURVATURE OF FIELD

EXPLANATION: There is a simple reason for the image plane of a flat wall to be curved; points further from the optical axis are further from the lens than the axial ray, and are imaged closer to the lens. The field curvature of a positive lens is shown in Figure 1. That of a negative lens is convex to the lens. It is the main reason that the edges of pictures are fuzzy, as the image plane lifts off the film.

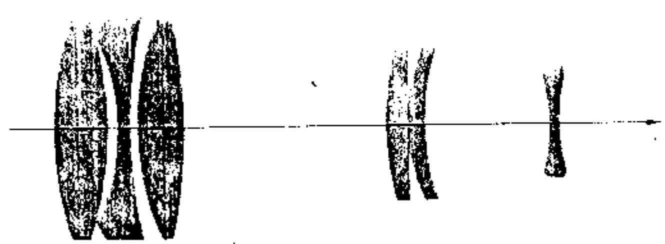


DEMONSTRATION: Look at the real image of the Johnson chart or a wall, focussed by the Big Lens on a groundglass and without it. Check to see if all the circles focus to the same plane on the groundglass.

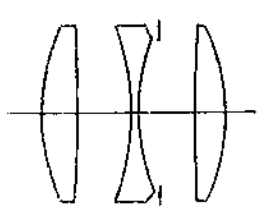
Observe the image without the groundglass. Does the flat object have a flat image?

CURE: The example shown in Figure 8 of **HANDOUT ABERRATION #3, ASTIGMATISM**, is an illustration of an <u>artificially flattened field</u>. Overcorrected astigmatism creates a flat image plane, but only because that is the locus of the circles of least confusion between the primary and secondary foci. Of this approach it has been said by nineteenth century photographers that they "relieved us of a blunder by substituting a sin".[1]

Since negative lenses have convex rather than concave fields, they can be used as <u>field flatteners</u>. Figure 2 shows a field flattening negative final element incorporated into a low f/# 16mm projector lens. The flattener is placed close to the image plane, so this design is not well suited for lenses for reflex cameras.



(b) 16 mm projection lens



The Cooke Triplet, which is the basis for many contemporary objectives, is an example of flat field design. See the **Handout, THE COOKE TRIPLET**.

Curvature of field is not affected by stop placement, nor does stopping down flatten fields. The depth of a curved field will grow. The curvature grows with the cube of off-axis angle, and the cube of object distance, making it formidable in wide-angle and macro designs. Choice of glass and lens shapes flattens the field.

The Eastman Kodak Company uses curvature of field creatively in their products. For their Carousel Slide Projectors, they offer two series of lenses, Ektanar-C and -FF.

For critical use, like seamless multi-projector presentations, the projected images need to be sharp edge to edge. The slides themselves are held perfectly flat in glass mounts. The -FF series of lenses image a flat slide onto a flat field.

But the majority of slides projected are mounted in cardboard or glass, and held only by their edges. The slide is concave to the emulsion side. The -C lens will image a Curved plane onto a flat one, which is the reverse of the usual picture-taking case.

Curvature of film is built into the 110 Pocket Instamatic Cartridge. By gently bowing the film stock, it becomes more rigid like a tape measure. The film will not wander in the image plane like its predecessor, the 126 Instamatic Cartridge. The image plane can be tailored to the non-flat film plane for sharp results.

REFERENCES

1. Rudolf Kingslake, A History of the Photographic Lens, Academic Press, San Diego, 1989, p.5.

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CURVATURE OF FIELD DEMONSTRATION

1. Focus the image of a wall or Johnson Tester onto a groundglass and watch the focal plane shift from center to edge.

REFERENCES

1. Rudolf Kingslake, A History of the Photographic Lens, Academic Press, San Diego, 1989, p.5.

THE SEVEN DEADLY ABERRATIONS
#5: DISTORTION

EXPLANATION: Magnification varies with object distance; closer objects are imaged larger than farther ones. A corner of a square is further from the optical axis of the lens than any otaher points on its side. But the corner may be closer to other parts of the lens, which may be the ones that are used in forming the image.

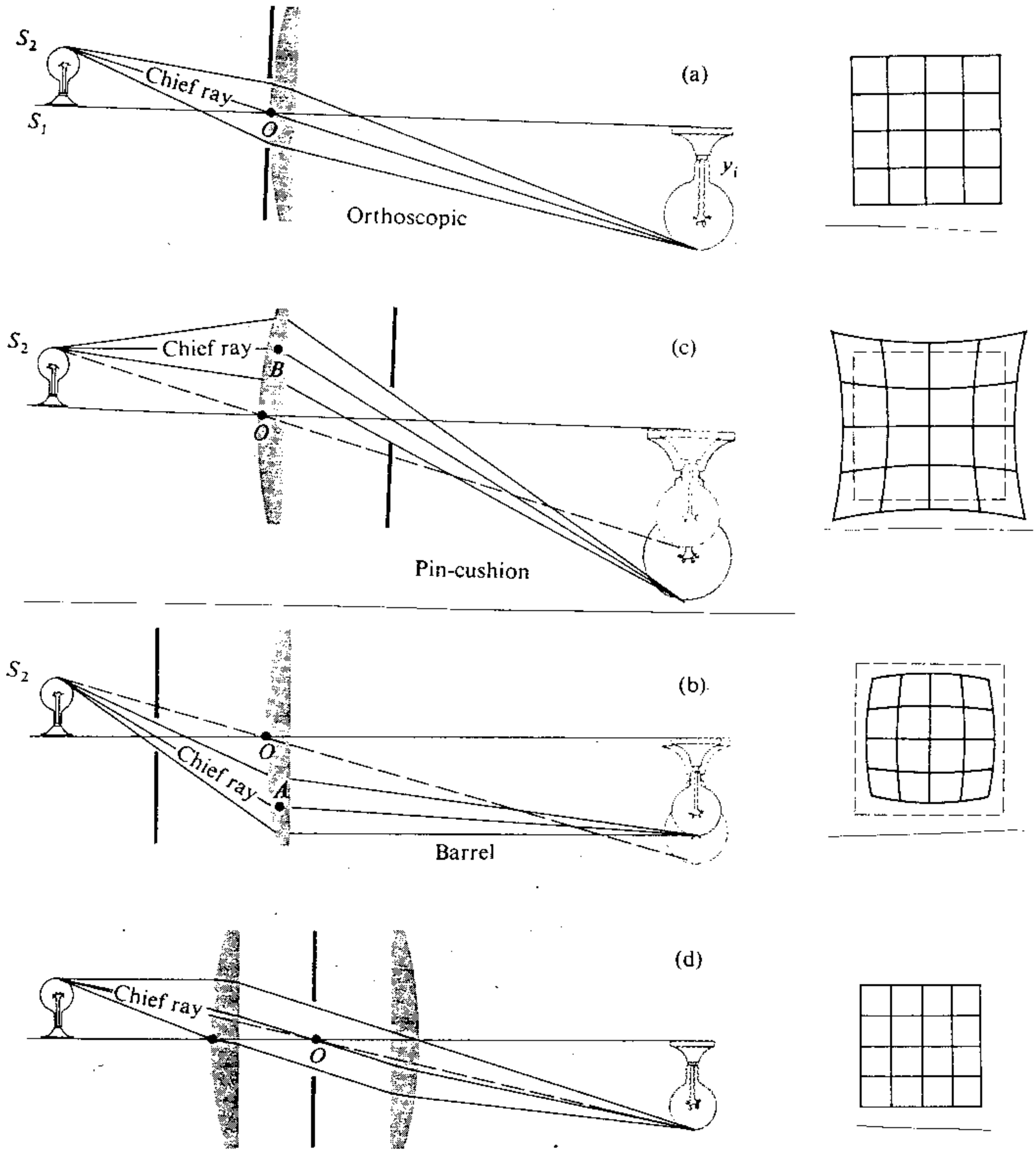
Figure 1a. shows an orthoscopic or right-viewing image of the light bulb. The aperture stop that is placed right against the lens is rather small, to that all the rays pass through the center of the lens and come to a common focus. On the right is imaged a square grid.

When the stop is in front of the lens, the chief ray from the top of the bulb is blocked, and the image of the bulb is formed by rays that had to travel further than the chief ray, and so they from a smaller than expected image. To the right of this ray trace is an image of the square grid, and since the corners are minified, they fall inside of their ideal location indicated by the dashed lines. The grid appears to be mapped onto curved surface, like a barrel, so that is another name for negative distortion.

With the stop behind the lens, the image of the top of the bulb is formed mainly by rays passing through the edge of the lens nearest to the bulb, which causes a magnification. The right side shows the image of the square grid again, with the enlarged corners flaring outward, and the sides pinched inward, like they had been jabbed, so positive distortion is AKA pincushion distortion.

The distorted shapes of the straight lines are not arcs of circles, but graph out the fact that the aberration depends on off-axis angle and object distance to the third power. The further away, the much more distorted.

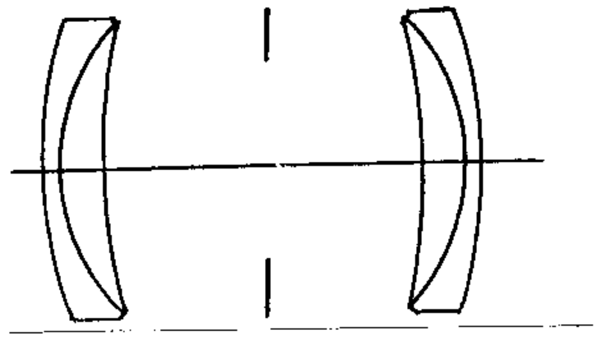
The d part of Figure 1 shows how to correct for distortion using the facts learned in 1b and 1c. The minification of the first lens is magnified by the second and vice versa. This symmetric approach has been used in many photographic applications. When the system is used at 1:1 imaging ratio, coma and transverse color drop out, making a lens excellent for Xerox copying. But even when used at normal picture taking ratios, this symmetric design works well.



S_2
Chief ray
S_1
O
(a)
y_i
Orthoscopic
S_2
Chief ray
B
O
(c)
Pin-cushion
S_2
(b)
O
Chief ray
A
Barrel
(d)
Chief ray
O

Figure 1

In 1866 photographic objectives using this principle were invented almost simultaneously by two different manufacturers. The Rapid Rectilinear was brought out by J. H. Dallmeyer and the Aplanat by Dr. H. A. Steinhheil, both of which positioned a pair of achromats on either side of a central stop. Figure 2 shows the basic scheme, with the inner positive elements of lower index than the outer negative meniscuses.



This style of lens was the top of the line for nearly sixty years and was copied by every manufacturer. Although of modest aperture by today's standards, f/6.2 to f/18, it was possible to use the same lens on enlarger as well as camera, or to use for unit magnification copy work.

A lens's image can be distorted yet still be perfectly sharp. Distortion is unavoidable in very wide angle optics, like fish-eye lenses.

Photographing an object through a lens with distortion, then using that same lens to project the image but with the lens turned around will give an undistorted projection. If figure 1b is the taking arrangement, with the stop on the side of the object, barrel distortion will be produced. When the barrelly distorted slide is projected with the stop on the outside of the same lens, (as the slide is now the object) the induced pincushion distortion straightens out the lines in the projected slide. It would be interesting to find out if the **OMNIMAX** type of theatres use this trick.

A single thin element gives the least distortion when the stop is at the optical center of the lens. (However, coma and astigmatism are best served by stops in front of the lens.) In a pinhole camera, the stop and the lens are one and the same, the pinhole itself, and therefore there is no distortion.

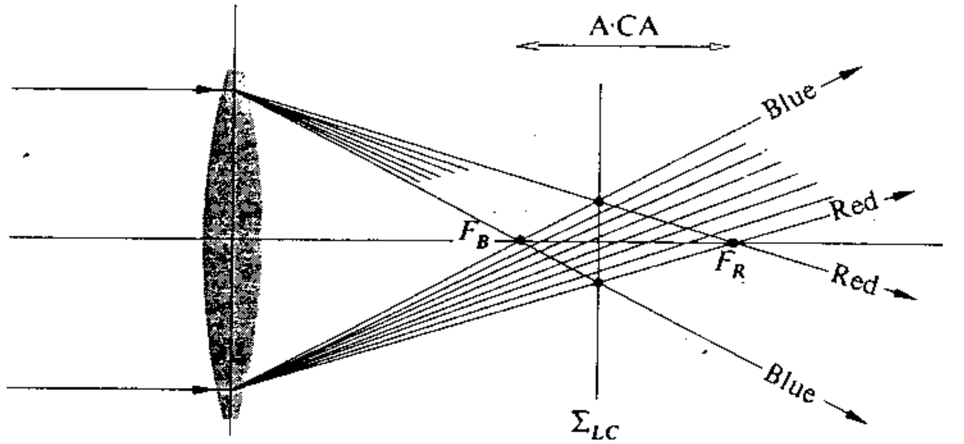
DISTORTION DEMONSTRATION

1. Focus the image of a grid or chicken wire on a groundglass. Put stops in front and behind the lens to show the change in shape of the distortion.

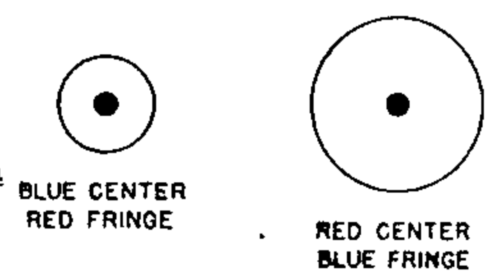
THE SEVEN DEADLY ABERRATIONS
#6 & #7: CHROMATIC ABERRATIONS

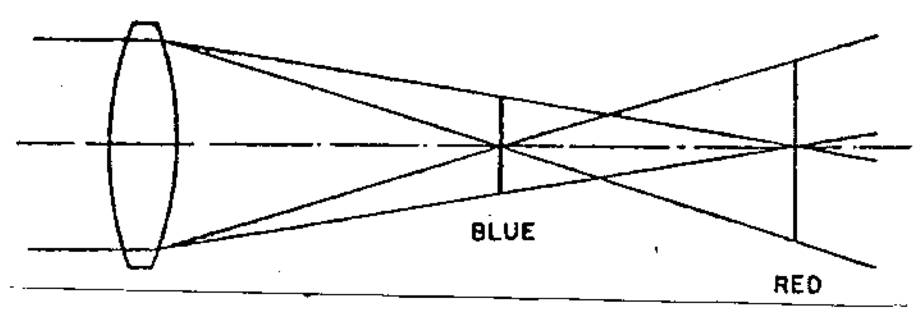
EXPLANATION: As can be seen from the Lensmaker's Formula, each color of the rainbow has its own focal length. (See the **Homework, LENSMAKERS' FORMULA.**) Since the blues have a higher index of refraction than the red end of the spectrum, they have a shorter focal length than the longer wavelengths, and there is a dispersed spectrum of images near the focal plane. The edge of the lens could be considered prismatic in shpae.

The ray trace of Figure 1 shows the consequences of this multiplicity of focal lengths. An infinitely far away point source of white light is focussed by the lens. The focal points of the rainbow are spread out along the optical axis.



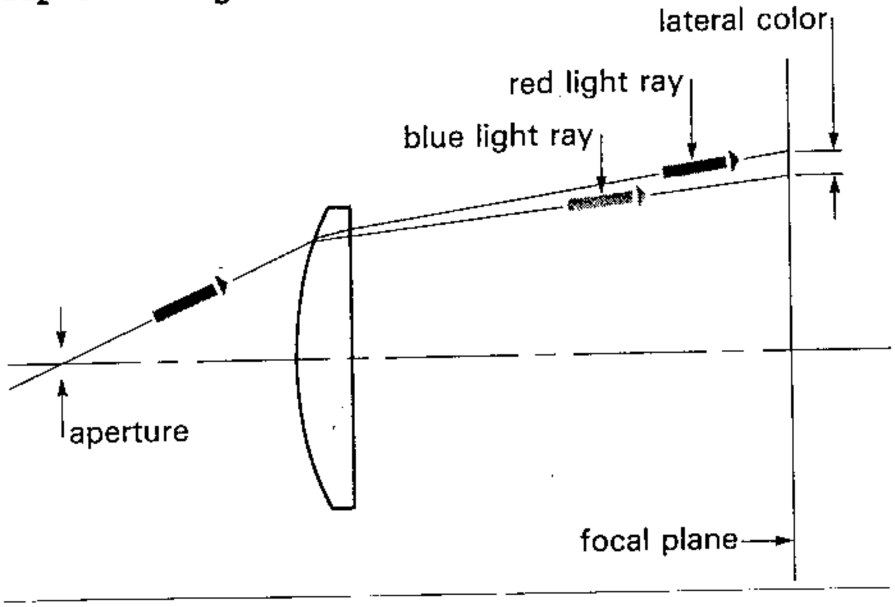
Images formed at the peripheries of the visible spectrum will have red or blue centers with vice-versa'ed outer fringes as in Figure 2. Focussing the colored images of a pinhole demonstrates the <u>Axial or Longitudinal Chromatic Aberration</u> and how it is measured. Put strong or saturated color filters in front of the lens, like Kodak Color Separation Filters, and watch the focal plane change.





The **Color Stereoscopic Phenomena** is a good demonstration of Axial Chromatic Aberration. Since the human eye is not achromatized, red and blue colors lying on the same plane of a graphic do not focus to the same plane on the retina. With the red focussing further from the eye lens, the ciliary muscles have to accomodate differently to focus the blue. Our eye tells the brain that the red and blue images must be at different depths since there is a change in accomodation in viewing the two clearly. But this is an optical illusion, since they both start on the same plane. Masterful use of this aberration can be utilized to make cheap 3-D image

To see Transverse Color (Figure 3) a backlit groundglass is imaged. A black spot is placed on it. A colored halo will be seen around the image of the black spot, due to the differences in image heights of the different wavelengths.



The Chromatic Aberrations are therefore #6 and #7, because they can be measured in two planes: Longitudinally or Axially, and Laterally or Transversely, terminology depending on textbook. The former refers to the location of the image's focal plane; the latter refers to the size of the image. Elimination of one does not necessarily mean freedom from the other.

Early photographers had to compensate for chromatic aberration by shifting the focal plane of their cameras from the visible rays image to the shorter wavelengths, blue and UV, which focussed closer to the film plane. (Early photographic materials were blind to green and red, and their peak sensitivity was in the UV.)

If two identical prisms are place in a row, the first would disperse the white light, and the dispersed light could be collected by the second and "undispersed" back into white. One prism's tip would be up, the other's is down.

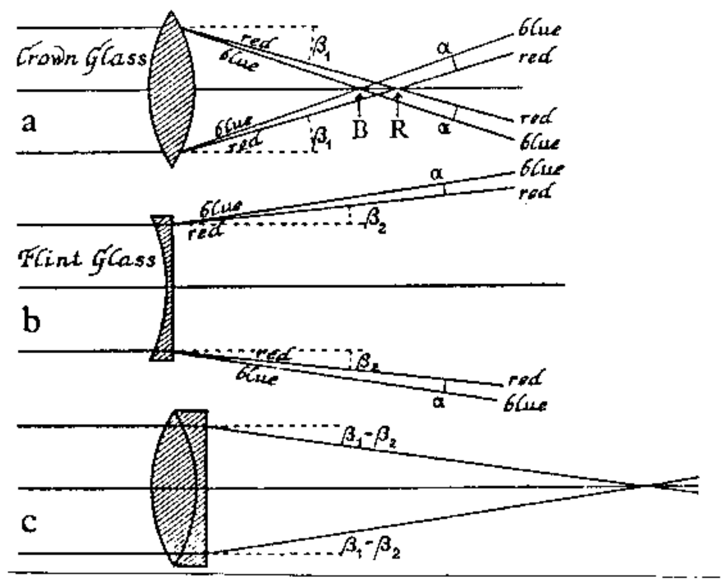
Identically shaped prisms of two different refractive index materials would have different bending angles for each color. The higher index material would bend light more, and make a larger rainbow.

Because the dispersive properties of glasses vary, it would be possible to design two prisms that have the same dispersive angle between red and blue, but have two different peak angles. (Think: which glass would have the smaller peak angle, the high or the low index?)

By putting the two different prisms in a row, the white light can be dispersed and then recombined just like the case of identical prisms. This is the basic principle of achromatization.

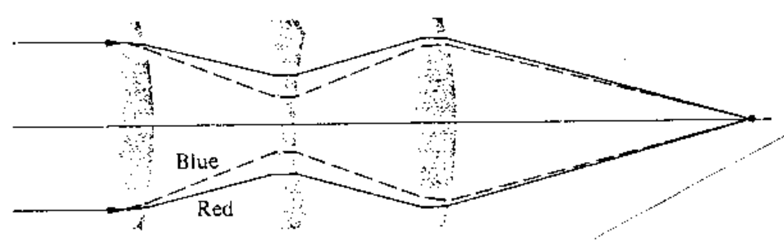
Figure 4 shows the construction of an achromatic doublet. the dispersive property of the positive convex lens separates the red and blue foci with an angle alpha between them. Notice the "tip" of the prism is pointing up.

The negative component is made of higher index flint glass, and is figured so that its dispersion angle is the same as the convex's. Notice that this lens's "prism" is tip down.



The combination then exhibits a common focus for the red and blue thanks to the dispersion compensation of their mating surfaces. The focal length of the combination is longer than that of the single convex lens, because the negative element takes away bending power. The trick in designing such a lens is that the dispersing power of the negative lens counteracts the dispersion of the positive without totally counteracting the focussing power.

This is why the elements of a cemented achromat cannot be made out of the same material, as converging power is cancelled as disperson is corrected. However air-spaced achromats can be designed using



the same glass throughout, as shown in Figure 5, a Cooke's Triplet.

Another way to eliminate chromatic aberration is to use monochromatic or reduced bandwidth of the light with filters. Things viewed under Mercury or Sodium vapor streetlights with their short spectra appear sharper than under white light.

The history of the Achromatic lens is an interesting one. Sir Isaac Newton in his first published paper in 1672 had stated that he believed that dispersion was the limiting factor in the performance of telescope objectives. The secret of achromatization was beyond Newton because he did not realize that different glasses have different variations of their indexes of refraction. Dispersion and refraction are not in a fixed ratio. For instance, the difference between the red and blue indexes for two glasses differ by two different per cents. See Table below.

	refractive index	
wavelength	BK-7	SF-11
457.9	1.54262	1.81596
632.8	1.51509	1.77862
difference	.02753	.03734
% change	1.7846%	2.0562%

But by 1733, an amateur optician, Chester Moor Hall, had discovered that mating a convex lens of low dispersion with a negative flint element of higher dispersion could result in a much less chromatically aberrated image.

But being a barrister by profession and not an optical scientist he tried to keep his discovery a secret. He used two different opticians to make the two separate elements. However, both these opticians sub-contracted the actual fabrication out to the same glass grinder and polisher.

When John Dollond, an optician, visited the same glassgrinder to pick up some of his own work, he noticed two mating lanses which were to be part of a new telescope objective. Once he saw them in action he realized their worth. He researched the relationship between refraction and dispersion for different glasses and applied for a patent. His paper, "An Account of some Experiments Concerning the Different Refrangibility of Light" and a published account of the mathematical theory of the achromatic lens earned him a Fellowhisp in the Royal Society of England.

Thanks to Dollond, lens manufacturers devoted more time to investigating and inventing new types of glass. Performance of telescopes were greatly improved, with microscopes finally becoming achrmatized by the end of the century. Early

photographic experimenters were urged to start with achromatic lenses, and the first manufactured camera, the Giroux Daguerreotype camera, sported an achromat.

The dispersive quality of different types of glasses are given by the glasses V-number or Abbe number. The refractive indexes of three wavelengths are compared in the calculation:

$$V = \frac{(n \ @ \ yellow \ -1\ )}{(n \ @ \ blue \ - \ n \ @ \ red)}$$

The lower the V-number, the more dispersive the glass. Glass Manufacturers' catalogs ususlly give the refractive index at the yellow, and the Abbe number to measure its dispersive property. (Yes it is possible to have the same n but different Abbe number, or vice versa. But the nature of glass does not make it easy to custom manufacture these properties.)

The wavelengths chosen were ones that could be found in the nineteenth century laboratory easily as Fraunhofer's lines in the solar spectrum:

red = 656.3 nm, yellow = 589.3 nm, blue = 486.1

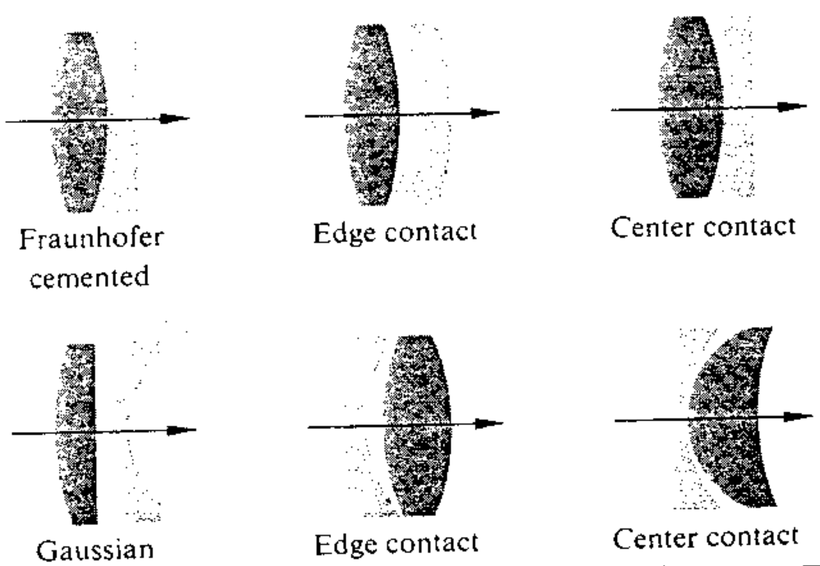
To design an achromatic doublet, the sum of the focal lengths and the products of their V-numbers should cancel out:

$$F1 \ @ \ yellow * V1 + F2 \ @ \ yellow * V2 = 0$$

Remember that to make an achromat, one lens has to have a negative focal length.

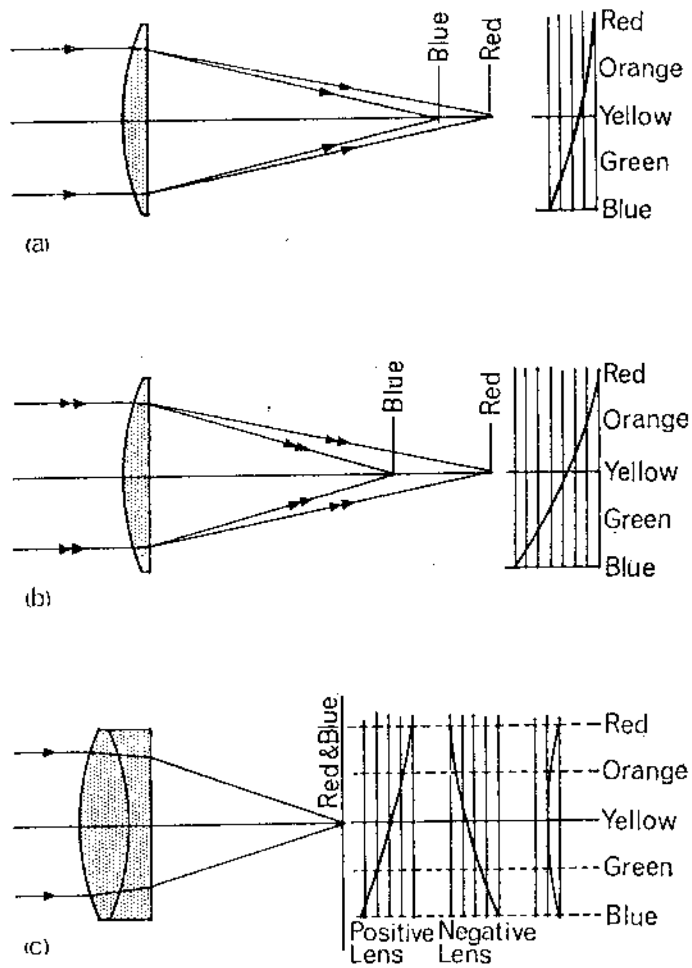
The focal length of the new lens is:

$$1/f = 1/f1 + 1/f2.$$



Achromats do not necessarily need to be cemented together. There are many choices for air-spaced achromats, too. Figure 6 shows examples, with the lighter toned element being the higher index element. Most refracting telescope objectives are air-spaced achromats. The thickness of the air gap is critical.

Here are two PCX lenses of two different glasses in Figure 7. The red focal lengths are the same; the blue are different. This is another illustration of the effect of the dispersive property of the Abbe number. (Think: which lens has the lower Abbe Number?)

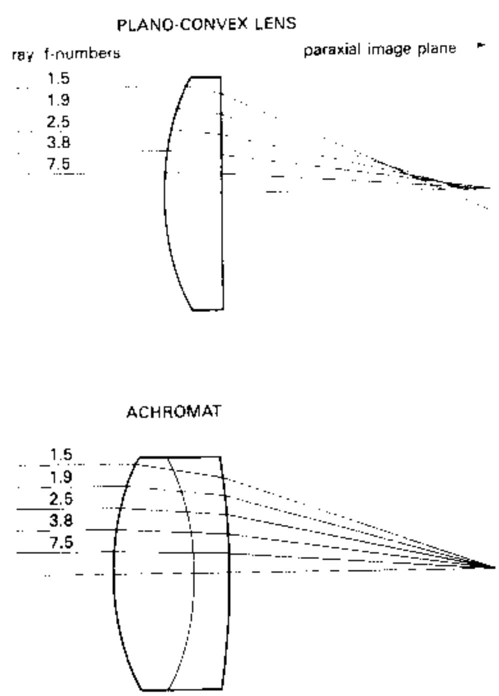


Achromatization refers to correcting the lens so that two wavelengths images are coincident, and hopefully the others are fairly close by, like in the example in the Figure 7 above. But the yellow part of the spectrum's image may not coincide with the blue and the red that the lens is corrected for. A secondary spectrum may arise, which produces green or magenta haloes around white objects.

Further correction gives apochromatized lenses.

Unlike the other aberrations, the chromatic one is not dependent on the aperture or the object's position in the field. Eliminating Transverse does not necessarily imply eliminating Longitudinal.

Although achromats perform better than the same focal length, same f/#, best form single element, Figure 8, they are not immune from all the other aberrations. Coma, Astigmatism, Curvature of Field and Distortion can all be present. One of the most popular lenses of the nineteenth century, Dallmeyer's Rapid Rectilinear, used a pair of achromats symmetrically mounted around a stop to combat coma and distortion. See Figure 2 in the **Aberration Handout #5**.



A curious application of achromatization is in prisms for anamorphic lens attachments. See also the **Handout, ANAMORPHIC LENSES**.

CHROMATIC ABERRATIONS

1. Look at Color Stereographic Illusion Slide and Jefferson Airplane graphic.

2. Light up a small hole drilled in metal in front of a halogen lamp and watch the blue and red foci change in the Longitudinal Chromatic Aberration. Use Color Separation Filters to show the difference.

3. Light up a groundglass with a black dot on it to see the Transverse Chromatic Aberration. Use filters again.